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Beyond Maximum Likelihood Model Estimation in Model-based Policy Search

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Table of Contents

Reinforcement Learning

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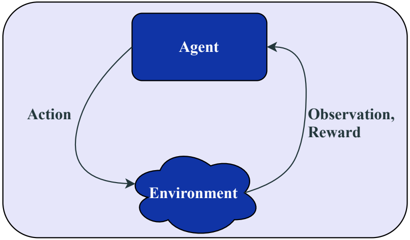
Overview

Solving Sequential Decision Making Problems

Sequential decision making is a core capability of intelligent agents.

Reinforcement Learning (RL) studies how an agent can learn to interact with an environment, guided by a reinforcement signal he wants to maximize.

No knowledge of the environment dynamics is assumed.



Markov Decision Processes

A Markov Decision Process (MDP) [Puterman, 2014] is described by a tuple $\mathcal{M} = (\mathcal{S}, \mathcal{A}, r, p, \mu, \gamma)$, where:

- $\mathcal{S}$ is the space of possible states
- $\mathcal{A}$ is the space of possible actions
- $r : \mathcal{S} \times \mathcal{A} \to \mathbb{R}$ is the reward function
- $p : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \to \mathbb{R}$ is the transition model
- $\mu : \mathcal{S} \to \mathbb{R}$ is the distribution of the initial state
- $\gamma \in [0, 1)$ is a discount factor

We assume r is known and uniformly bounded by $|r(s, a)| \leq R_{\max} < +\infty$. The behavior of an agent is described by a policy $\pi : \mathcal{S} \times \mathcal{A} \to \mathbb{R}$.

Value Functions and the RL objective

Given a state-action pair (s, a) we define the action-value function [Sutton and Barto, 2018], or Q-function, by using the dynamic-programming-based Bellman equation:

$$Q^{\pi,p}(s,a) = r(s,a) + \gamma \int_{\mathcal{S}} p(s'|s,a) \int_{\mathcal{A}} \pi(a'|s')\, Q^{\pi,p}(s',a') \mathrm{d}s' \mathrm{d}a'$$

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and the state-value function, or V-function, as:

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The goal of the agent is to find an optimal policy π^*, maximizing the expected return:

$$J^{\pi,p} = \mathbb{E}_{s_0 \sim \mu} \left[V^{\pi,p}(s_0) \right], \quad \pi^* = \arg\max_{\pi} J^{\pi,p}$$

Successes in Model-free RL

Most RL approaches do not explicitly learn about the transition model p.

Biggest successes in model-free RL in the last years were in games. For instance, super-human performance was reached in:

- ATARI games [Mnih et al., 2015]
- Go [Silver et al., 2016, Silver et al., 2017]
- Starcraft [Vinyals et al., 2019]

Shared traits: very efficient simulator available, no safety, transfer or data efficiency concerns.

In others words, not easy to adapt these methods to the real world.

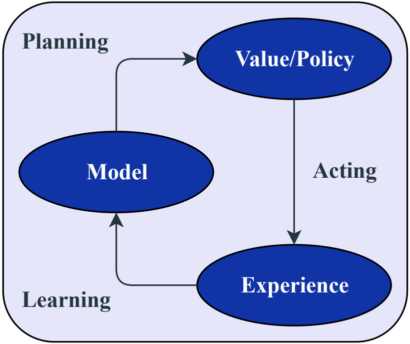
Reinforcement Learning

Model-based Reinforcement Learning

The model-based loop

Model-based Reinforcement Learning (MBRL) uses estimated models of the dynamics of the environment to learn a policy. Synonyms: world model, forward model, (just) model.

The policy is obtained by planning with the learned model.



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- Decision-aware. Leverage knowledge about task, policy or learning algorithm to decide which one of the observed transitions are more important. E.g. Minimize error on Bellman operator in value-based methods [Farahmand et al., 2017]:

$$c\left(\hat{p}, p; Q\right)(s, a) = \left| \int \left[p\left(s'|s, a\right) - \hat{p}\left(s'|s, a\right)\right] \max_{a'} Q(s', a') \mathrm{d}s' \right|$$

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RESEARCH GOAL: learn the model of the dynamics that is optimal for improving a policy by using its gradient.

Policy Gradients

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A gradient ascent approach to policy search

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- The performance on the task J is therefore a function of $\boldsymbol{\theta}$
- We can differentiate it w.r.t. the parameters of $\pi_{\boldsymbol{\theta}}$
- Then, policy can be improved with an update of the kind:

$$\boldsymbol{\theta}^{k+1} = \boldsymbol{\theta}^k + \alpha \nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta}),$$

where α is a small step-size (a.k.a. learning rate)

Policy Gradients

A taxonomy of Policy Gradients

Model-Free Gradient

Let p be the transition model of an MDP, Π_Θ a parametric space of stochastic and differentiable policies, $\pi \in \Pi_\Theta$.

Definition (Model-Free Gradient)

$$\nabla_{\boldsymbol{\theta}}^{\mathrm{MFG}} J(\boldsymbol{\theta}) = \frac{1}{1-\gamma} \int_{\mathcal{S}} \int_{\mathcal{A}} \delta_{\mu}^{\pi,p}(s,a) \nabla_{\boldsymbol{\theta}} \log \pi(a|s) Q^{\pi,p}(s,a) \mathrm{d}s \mathrm{d}a.$$

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Unbiased estimators are available.

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- It can reduce variance at the cost of the bias introduced by an estimated model

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It uses trajectories from the real environment model p, but solves credit assignment by using the estimated model $\hat{p}$. Examples: SVG(∞) [Heess et al., 2015], MVE [Feinberg et al., 2018].
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Advantages:

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- A compromise in bias and variance w.r.t. MFG and FMG

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Model-Value-based Gradient $\Rightarrow$ Compromise on bias/variance

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Gradient-Aware Model-based Policy Search

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Analysis of the Model-Value-based Gradient

An upper bound on the MVG bias

Proposition

Let $q \in [1, +\infty]$ and $\widehat{p} \in \mathcal{P}$. If $\|\nabla_{\boldsymbol{\theta}} \log \pi(a|s)\|_q \leq K$ for all $s \in \mathcal{S}$ and $a \in \mathcal{A}$, then, the L^q-norm of the difference between the policy gradient $\nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta})$ and the corresponding MVG $\nabla_{\boldsymbol{\theta}}^{\mathrm{MVG}} J(\boldsymbol{\theta})$ can be upper bounded as:

$$\|\nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta}) - \nabla_{\boldsymbol{\theta}}^{\mathrm{MVG}} J(\boldsymbol{\theta})\|_q \leq c_1 \sqrt{\mathbb{E}_{s,a \sim \delta_\mu^{\pi,p}} \left[D_{KL}(p(\cdot|s,a) \| \widehat{p}(\cdot|s,a)) \right]}.$$

This proposition justifies maximum likelihood model estimation for MVG-based approaches.

The importance of the error on state-action couple (s, a) only depends upon its visitation $\delta_\mu^{\pi,p}(s, a)$.

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$$\begin{aligned}\|\nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta}) - \nabla_{\boldsymbol{\theta}}^{\mathrm{MVG}} J(\boldsymbol{\theta})\|_q &\leq c_2 \sqrt{\mathbb{E}_{s,a \sim \eta_\mu^{\pi,p}} \left[D_{KL}(p(\cdot|s,a) \| \widehat{p}(\cdot|s,a)) \right]} \\ &\leq c_1 \sqrt{\mathbb{E}_{s,a \sim \delta_\mu^{\pi,p}} \left[D_{KL}(p(\cdot|s,a) \| \widehat{p}(\cdot|s,a)) \right]}.\end{aligned}$$

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The distribution under which model mismatch is measured becomes:

$$\eta_\mu^{\pi,p}(s,a) = \frac{1}{Z} \int_{\mathcal{S}} \int_{\mathcal{A}} \delta_\mu^{\pi,p}(s',a') \left\| \nabla_{\boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(a'|s') \right\|_q \delta_{s',a'}^{\pi,p}(s,a) \mathrm{d}s' \mathrm{d}a'$$

A decision-aware weighting factor leads to a better bound.

Gradient-Aware Model-based Policy Search

The Algorithm

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2. Computing the value function

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Gradient-Aware Model-based Policy Search (GAMPS) has 3 steps:

1. Learning the model
2. Computing the value function
3. Estimating the policy gradient

Learning the model

Gradient-Aware loss function

$$\widehat{p} = \arg\max_{\overline{p} \in \mathcal{P}} \frac{1}{N} \sum_{i=1}^{N} \sum_{t=0}^{T_i - 1} \omega_t^i \log \overline{p}\left(s_{t+1}^i | s_t^i, a_t^i\right)$$

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Gradient-Aware Weights

$$\omega_t^i = \gamma^t \rho_{\pi/\pi_b}(\tau_{0:t}^i) \sum_{l=0}^{t} \left\| \nabla_{\boldsymbol{\theta}} \log \pi(a_l^i | s_l^i) \right\|_q$$

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Important transitions:

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Important transitions:

- Early transitions

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Important transitions:

- Early transitions
- Likely transitions under π
- Transitions at the end of trajectories with large cumulative score magnitude

Computing the value function

To compute $Q^{\pi,\hat{p}}$, we should carry out policy evaluation.

We propose the following Monte-Carlo approach for continuous environments.

Evaluation via Monte-Carlo Imagination Rollouts

$$\widehat{Q}(s,a) = \frac{1}{M}\sum_{j=1}^{M}\sum_{t=0}^{T_j-1}\gamma^t r(s_t^j, a_t^j), \quad \tau^j \sim \zeta_{s,a}^{\pi,\widehat{p}}.$$

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Advantages in avoiding explicit Q-function approximation:

- No regression targets
- No choice for model class

Estimating the policy gradient

Policy Gradient estimator

$$\widehat{\nabla}_{\boldsymbol{\theta}} J(\boldsymbol{\theta}) = \frac{1}{N} \sum_{i=1}^{N} \sum_{t=0}^{T_i - 1} \gamma^t \rho_{\pi/\pi_b}(\tau^i_{0:t}) \nabla_{\boldsymbol{\theta}} \log \pi(a^i_t | s^i_t) \widehat{Q}(s^i_t, a^i_t).$$

Putting all together

Gradient-Aware Model-based Policy Search

Input: Trajectory dataset $\mathcal{D}$, behavior policy π_b, initial parameters $\boldsymbol{\theta}_0$

for $k = 0, 1, ..., K-1$ do

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Gradient-Aware Model-based Policy Search

Theoretical Analysis

Goal of the theoretical analysis

Learning theory analysis using tools from [Cortes et al., 2013].

Objectives:

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- Highlight the important elements in approximation/estimation errors

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- Show that choosing a simple model class can be wise
- Justify the intuition behind gradient-aware MVG estimation

Finite-sample Bound

Theorem

For any $\delta \in (0,1)$, with probability at least $1-4\delta$ it holds that[a]:

$$\left\| \widehat{\nabla}_{\boldsymbol{\theta}} J(\boldsymbol{\theta}) - \nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta}) \right\|_q \leq \underbrace{c_2 \inf_{\overline{p} \in \mathcal{P}} \sqrt{\mathop{\mathbb{E}}_{s,a \sim \eta_\mu^{\pi,p}} \left[D_{KL}(p(\cdot|s,a) \| \overline{p}(\cdot|s,a)) \right]}}_{\text{approximation error}} + \underbrace{\mathcal{O}\left(\sqrt{\frac{v}{N}}\right)}_{\text{estimation error}}$$

[a] N: num. of trajectories, v: $\mathcal{P}$ pseudo-dimension, d: gradient dimensionality

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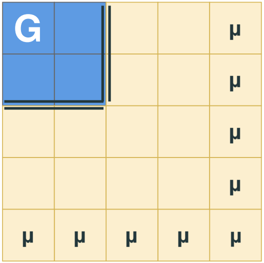
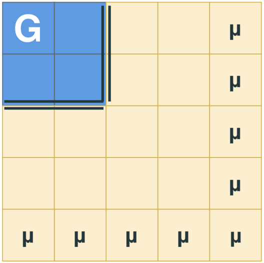
Gradient-Aware Model-based Policy Search

Experimental Analysis

Two-areas Gridworld - Description





- Swapped action effect in two areas

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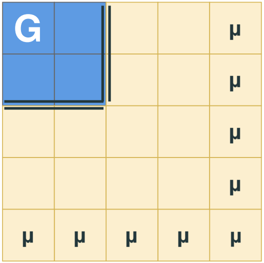

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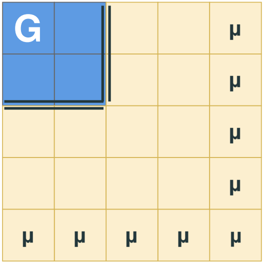

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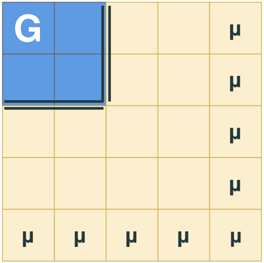

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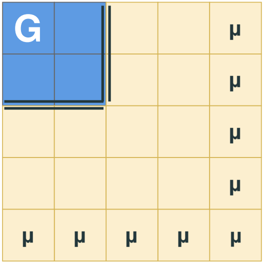

- Swapped action effect in two areas
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- Reward of -1 everywhere but in G
- One-way wall
- Batch setting (one time data collection)

Two-areas Gridworld - Policy and Model class

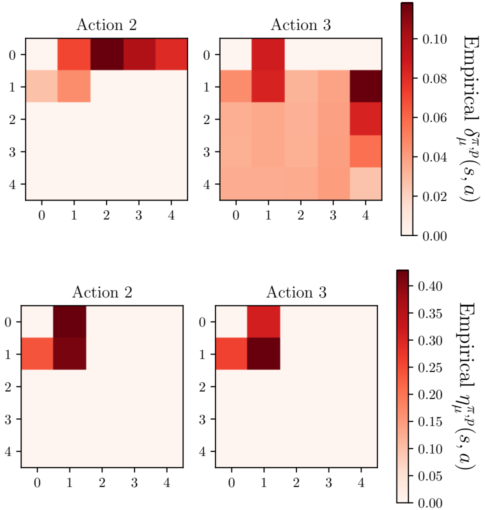
Policy

- Boltzmann (categorical) distribution over actions
- Deterministic in lower part, randomly initialized in upper

Model

- Linear in the sole action (equivalent to a lookup table)
- Able to represent only one of the two environment parts

Two-areas Gridworld - Difference in ML and GAMPS weights



Two-areas Gridworld - Gradient Approximation Experiment

Table 1: Estimation performance on the gridworld environment comparing Maximum Likelihood estimation (ML) and our approach (GAMPS). 1000 training and 1000 validation trajectories per run. Average results on 10 runs with a 95% confidence interval.

Approach	$\widehat{p}$ accuracy	$\widehat{Q}$ MSE	$\widehat{\nabla}_{\boldsymbol{\theta}} J$ cosine similarity
ML			
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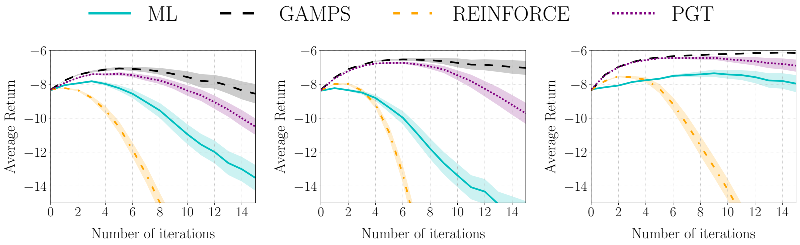
GAMPS	0.357 ± 0.004	633.835 ± 12.697	1.000 ± 0.000

Two-areas Gridworld - Policy Improvement Results



(a) 10 trajectories (b) 50 trajectories (c) 100 trajectories

Figure 1: Average return on the Two-areas gridworld with different dataset size. ML is the same as GAMPS but using maximum likelihood model estimation (20 runs, mean $\pm$ std).

Minigolf (Continuous states/actions) - Policy Improvement Results

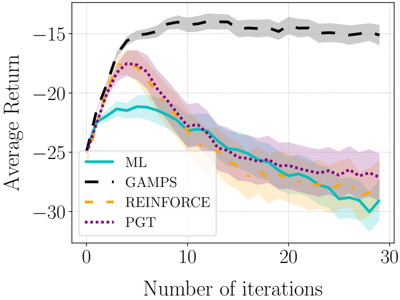


Figure 2: Average return using a 50 trajectories dataset on the minigolf environment (10 runs, mean ± std).

Conclusions and Future Work

Recap

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- We analyzed the Model-Value-based Gradient
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- We built the GAMPS algorithm based on this intuition
- We validated the theoretical analysis with experimental evidence

Future Work

Possible extensions and related research directions:

Future Work

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- Online extension

Future Work

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- Online extension
- Different methods for computing $Q^{\pi,\hat{p}}$

Future Work

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- Online extension
- Different methods for computing $Q^{\pi,\hat{p}}$
- Estimation of the reward function

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- Online extension
- Different methods for computing $Q^{\pi,\hat{p}}$
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- Deeper theoretical analysis

Future Work

Possible extensions and related research directions:

- Online extension
- Different methods for computing $Q^{\pi,\hat{p}}$
- Estimation of the reward function
- Deeper theoretical analysis
- Other gradient-aware MVGs (e.g., inspired by SVG)

In Model-based RL

Maximum likelihood is an agnostic way to learn a model, but better loss functions exist when more information is available.

More generally - The meta-learning perspective

If a system uses different internal modules, the learning algorithm of a module can benefit from the knowledge about the learning algorithm of another.

The End[1]

Thank you for the attention!

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