State of the Art on: Multi-Robot Coverage

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1. Introduction to the research topic

The Multi-Robot Coverage topic lies in the area of Multi-Robot Systems, which is a research area at the intersection of Artificial Intelligence and Autonomous Mobile Robotics. It studies methods and techniques to obtain advantages from the coordination and cooperation of multiple robots and to enable the execution of tasks that are inherently executable only by multiple agents. It is a broad field that has many different aspects, that span from modeling and control, to planning and decision-making, and to all the technologies like software platforms, operating systems, and simulation tools that enable further developments and practical applications of Multi-Robot Systems.

Publications in the Multi-Robot Systems area target general Artificial Intelligence (AI) and Robotics venues, and a few more specific venues. The specific topic of the publication drives the choice of the venues to consider. According to the people in the area, the rankings, and the most common metrics, the most important conferences in the generic AI area are the Association for the Advancement of Artificial Intelligence conference, AAAI, and the International Joint Conference on Artificial Intelligence, IJCAI. Specific for multi-agent research (which includes the multi-robot area) there is the International Conference on Autonomous Agents and Multiagent Systems, AAMAS. The top Robotics conferences are ICRA (International Conference on Robotics and Automation), IROS (International Conference on Intelligent Robots and Systems), and RSS (Robotics: Science and Systems). Regarding the journals, there is AIJ (Artificial Intelligence Journal), that is the most important one in the AI area, and for the Robotics area: T-RO (IEEE Transactions on Robotics) and IJRR (The International Journal of Robotics Research). Among the most important venues with a narrower target on the specific area of Multi-Robot Systems, there are two conferences, DARS (Distributed Autonomous Robotic Systems) and MRS (Multi-Robot and Multi-Agent Systems), and two journals, AURO (Autonomous Robots) and RAS (Robotics and Autonomous Systems), which cover the area of autonomous robotics.

1.1. Preliminaries

The Multi-Robot Coverage topic makes use of many of the tools developed and used in the general AI area. In particular, a popular mathematical tool is graph theory, mainly because coverage problems are based on the use of maps and one of the most common choices to represent them is using graph. As a consequence, a good proficiency on graph/tree exploration algorithms is required to approach this topic. Mathematical programming and Operations Research should be well known since, often, optimization problems need to be solved and decisions have to be made. A specific mathematical formulation often used is the Traveling Salesman Problem (TSP) and, its extension, the multi Traveling Salesmen Problem (mTSP). They model settings in which all the nodes of a graph should be visited in an optimal way, w.r.t. some time or distance metrics, by one or multiple agents. Knowing computational complexity theory helps in the analysis of the problems and in devising better algorithms. An approach that is often used to devise efficient algorithms is dynamic programming.

From the technological point of view, there are only few tools that are used and they are mainly mathematical programming solvers (e.g., CPLEX, Concorde TSP Solver), and simulation tools (e.g., Stage, ARGoS, Gazebo, MRESim) used in conjunction with a robotic middleware (e.g., ROS) which can then be used for the implementation on real robots.

1.2. Research topic

Coverage is the problem of finding optimal paths, w.r.t. time or distance metrics, that allow agents equipped with covering tools of finite size to completely cover a known environment. Formally, it is defined as the planning of a

path in the environment such that, when followed by the robots, all the points fall under the covering tool of at least one robot, the constraints of the environment, i.e., obstacles, are considered, and the path is optimal w.r.t. a certain metric. Coverage is a task related to exploration, where the map is not known, and in the literature they are referred either as I introduced them or as 'coverage with known map' and 'coverage with unknown map'. Here and in the following, 'coverage' will have the meaning of coverage with known map.

The multi-robot coverage topic has important aspects of both theoretical (e.g., complexity of algorithms) and practical (e.g., autonomous ship hull inspection [12]) nature and related to both the AI (e.g., high-level planning) and the robotics (e.g., localization and mapping) domain. Hereinafter, I will mainly consider problems and works of theoretical nature in the AI domain.

Multi-Robot Coverage is an approach to coverage that exploits the usage of multiple robots to reduce the coverage time, to offer robustness (e.g., supporting the loss of a robot), and to enable coverage in special settings or under particular constraints [13].

This increased potentiality comes with an increased algorithmic complexity due to methods that must take into account coordination and relations between different robots. Indeed, while there exists a polynomial-time coverage algorithm for the single robot case as shown by [26], finding a minimum travel time solution for the general multiple robot setting is an NP-hard problem [22].

The coverage problem, and as a consequence the advantages brought by the multiple-robot approach, is driven by the practical need to physically pass over all the specified area (for example in applications like lawn mowing, floor cleaning or robotic de-mining [8]), to just gather data about the environment (e.g., water quality monitoring [24]), or for search and rescue applications.

Because of the NP-hardness of the Multi-Robot Coverage problem, one of the main problems in this field is the development of approximated algorithms and the identification of specific settings with constraints that enable more efficient algorithms.

2. MAIN RELATED WORKS

In this section, I first describe which are the main different directions of research in the topic of Multi-Robot Coverage and then I proceed to review the most representative works. Due to the differences of terms adopted in the different sub-domains and applications, I need to first explain the meaning of two terms. In the following, the term 'agents' will be used in place of agents, robots, or salesmen. Similarly, 'nodes' will be used to mean nodes or cities.

2.1. Classification of the main related works

A first classification dimension regards the scope of the research and we can differentiate between works more general and with a theoretical impact and works much more specific and usually related to a particular application.

We can then classify works with respect to the settings and to the solution approaches they investigate. Some works provide offline algorithms and solutions, others consider an online approach. In the latter case, communication and online coordination among agents are sometimes part of the settings and the algorithms.

We can identify two types of works with respect to the perception capabilities of the agents: in some works only the current node is part of the perception; in other works, the agent is able to perceive the environment, usually with a finite range.

At last, we can highlight the distinction between centralized algorithms, in which a single computational unity elaborates a strategy that is then communicated to all the agents, and distributed algorithms (typical of swarm robotics), in which there are multiple computational unities and a global strategy emerges as a result of the interaction between agents.

Multi-Robot Systems is a topic which has grown over the years but its history is not very long. As a consequence, in this domain there are not issues that could be labeled as fully assessed, unless very specific and narrow problems are considered. A fully developed theory is the one we inherit from Operations Research regarding the formulations

and approaches to TSP and especially mTSP. Moreover, a related matter is the computational complexity of the algorithms for TSP and related: unless $P = NP$, they are well-known for being intractable in the general case.

A first open research problem is directly related to this last issue: finding polynomial time algorithms for the mTSP that provide good approximation factors. Then there is a good amount of research that tries to address and/or exploit peculiarities of specific settings: communication constraints, presence of a base station, presence of a cable that constraints the movements. In these cases, there are not clear and optimal solutions yet and there is still much to investigate in this direction. Similarly, the structure of the environment is a feature that has an impact on the algorithms we can use. Many different settings are possible (e.g., grids, trees) and many of them have been explored, but the research is far from complete.

2.2. Brief description of the main related works

[4] presents an overview of formulations and solutions of the mTSP. The general definition of mTSP is the following: given a set of nodes, let there be m agents located at a single initial node. The remaining nodes are called 'intermediate nodes'. The mTSP consists of finding tours for all the m agents, which all start and end at the initial node, such that each intermediate node is visited exactly once and the total cost of visiting all nodes is minimized. The cost metric can be defined in terms of distance or time. In the area of Multi-Robot Systems we are usually interested in minimizing the total time of execution.

An important theoretical result is that every mTSP can be approximately solved through a corresponding TSP formulation [10, 25, 19, 5]. The corresponding formulation is obtained creating m copies of the original depot, each connected to the other nodes exactly as the original depot, and manipulating the cost matrix. The TSP solution obtained on this new graph is forced to have m tours: the TSP path will go through each copy of the original depot m times. If we "cut" the TSP path every time it passes through a copy of the original depot, we obtain m paths, each of them starting from a copy of the depot and ending in a copy of the depot. These m paths are the mTSP (approximated) solution.

A common approach for coverage problems in multi-agent settings is to preliminary group the nodes into m clusters[1], so that each cluster represents a set of adjacent nodes that can be visited by a single agent whose path can be optimized as in a standard TSP [14, 17, 7].
This idea of clustering is very used mainly because of the computational advantages: the search space of a routing problem with N nodes is a function of $N!$. A decomposition of the problem into k clusters means that the average number of nodes in a cluster is N/k. Therefore the search space for each cluster is $(N/k)!$ and the total search space is a function of $k \times (N/k)!$, which is much lower than $N!$ [7]. Several clustering algorithm have been discussed in [2].

[14] approaches the mTSP in two steps: first, they create k clusters using k-means algorithm with k equals to the number of agents. In the second step, they solve each cluster independently using ACO (Ant Colony Optimization) algorithm, which is a swarm intelligence optimization method inspired by natural behavior of ants.
In [17] a three-stage approximation to solve mTSP is proposed: in the first stage the mTSP is converted into a set of TSPs using k-means clustering algorithm. The second stage builds the initial tour for each agent with Shrink Wrap algorithm and, in the third stage, the initial tours are optimized.
For all the settings in which the completion time should be minimized, a very interesting work is the one by [7]. In their work, they address the mTSP problem with the objective of balancing the workload among the agents and finding out the optimal number of agents required to cover the set of nodes. They first define the cluster length as the distance traveled by an agent to cover the cluster and return to the depot. Then, they estimate the optimal number of clusters k, with k inside a predefined range, computing the coefficient of variation $\left(CV = \frac{\text{variance of clusters lenght}}{\text{average cluster length}} \right)$ of the cluster length.
A similar idea of workload balancing is discussed by [18]. Here the problem is approached imposing lower and upper limits to the travel times.

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[1]Clustering is a technique to divide a set of objects into groups called clusters, in such a way that objects in the same cluster are similar (according to some metric) to each other and objects in different clusters are dissimilar (according to the same metric). Proximity and distance measures can be used as a similarity measure [7, 2].

The mTSP can be considered as a relaxation of the VRP (Vehicle Routing Problem), with the capacity restrictions removed. This implies that all the formulations and solutions for the VRP are also valid for the mTSP [4]. The VRP is the problem that asks for the optimal set of paths for a number of vehicles in order to deliver goods to a set of costumers considering that each vehicle has a capacity constraint. The vehicles are the equivalent of the agents of the mTSP and the costumers are the correspondent of the nodes. Therefore, the equivalence is obtained setting the capacity of each vehicle to infinity.

[28] studies the VRP with the objective of minimizing the makespan, i.e., the maximum length of any vehicle route. This is the equivalent of minimizing the maximum traveling time. This is done through a mixed-integer linear programming formulation in which they minimize the maximum length over all the routes.

2.3. Discussion

mTSP is a generalization of the TSP problem and it is NP-hard. Hence, we should seek approximation algorithms because the optimal solution should be considered unreachable in general. As pointed out in [15], when actual robotics applications are considered, it is very reasonable to aim for fast approximation algorithms due to the presence of motion constraints [21]. Indeed, in these cases, when kinematic and dynamic assumptions are considered, the solution provided by the high-level algorithm is often unfeasible and should be adapted, or an algorithm that takes into account these constraints should be devised, see for example [23].

[15] provides an algorithm with a constant approximation factor of 2 for the Generalized, Multiple Depot, Multiple Travelling Salesman Problem (GMTSP), in the case of symmetric costs and with the triangle inequality satisfied. [11] provides, for the mTSP, a tour-splitting heuristic that yields an approximation factor of $\frac{5}{2} - \frac{1}{m}$ relying on the $\frac{3}{2}$ TSP approximation by [9]. An open question is whether better approximation algorithms can be found.

In recent years, there has been a lot of work intended to approach more practical settings, which in general add to the problem constraints of different types. For example, there are many works that consider the presence of a constraint of communication, of which [1] provides a survey of common approaches. Communication is a feature that is needed for all the algorithms and approaches that are based on online coordination among agents. As explained in the survey, in real-world settings, the communication cannot be assumed to be without limitations and this constrains the agents to share information only with teammates inside their communication ranges. Hence, achieving a good level of coordination becomes problematic.

Related settings are those in which there is a base station (BS) to which the agents should be connected, either continuously or at intervals. In this case, the common approach is to have agents that work as relay nodes to ensure connectivity to farther agents [16]. This setting is further explored by [20] that consider a moving base station.

Then there is a direction which instead is intended to provide tighter bounds: making assumptions on the structure of the environment gives us the possibility to exploit specific features and characteristics to have algorithms, tailored on the specific setting, that provide tighter bounds than what we can expect in the generic case. Typical settings involve grid graphs [27], usually extended to grids with holes [3], or environments that can be modeled as trees, as in [6]. Even if these works are not directly in the field of coverage but in the broader domain of Multi-Robot Path Planning (MPP), the settings they introduce can be adopted for the coverage problem. This direction is yet to be explored and it is not yet clear which are restrictions on the environment that will allow for good improvements.

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