Research Project Proposal:

Efficient Solutions for Adversarial Team Games

Federico Cacciamani
Federico.cacciamani@mail.polimi.it
CSE Track

POLITECNICO MILANO 1863

HONOURS PROGRAMME HP-SR in Information Technology

Outline

1. Introduction to Algorithmic Game Theory
2. Preliminaries
3. State of the art
4. Project proposal

Outline

1. **Introduction to Algorithmic Game Theory**
2. Preliminaries
3. State of the art
4. Project proposal

Algorithmic (Game Theory)

*“**Game theory** is the name given to the methodology of using mathematical tools to model and analyze situations of interactive decision making. These are situations involving several decision makers (called players) with different goals, in which the decision of each affects the outcome for all the decision makers.”*

M. Maschler, E. Solan, S. Zamir. “Game Theory”. 2013

- Algorithmic Game Theory is the area at the intersection between **Game Theory** and **Computer Science**

Recreational Games

Recreational Games

Recreational Games

Recreational Games

Recreational Games

Recreational Games

IBM

Carnegie Mellon University

Recreational Games

IBM

Carnegie Mellon University

Google DeepMind

Recreational Games

IBM

Carnegie Mellon University

Google DeepMind

StarCraft II

Recreational Games

IBM

Carnegie Mellon University

Google DeepMind

STARCRAFT II

Google DeepMind

Potential Real-World Applications

- **Physical Security**: Strategic organization of the available resources

- **Car Races**: Coordination of strategies among team members

Outline

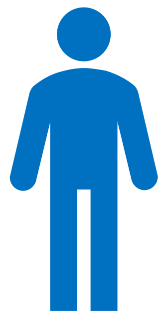
1. Introduction to Algorithmic Game Theory
2. **Preliminaries**
3. State of the art
4. Project proposal

Game Tree

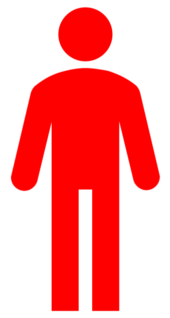
Game Tree

Player 1 Player 2

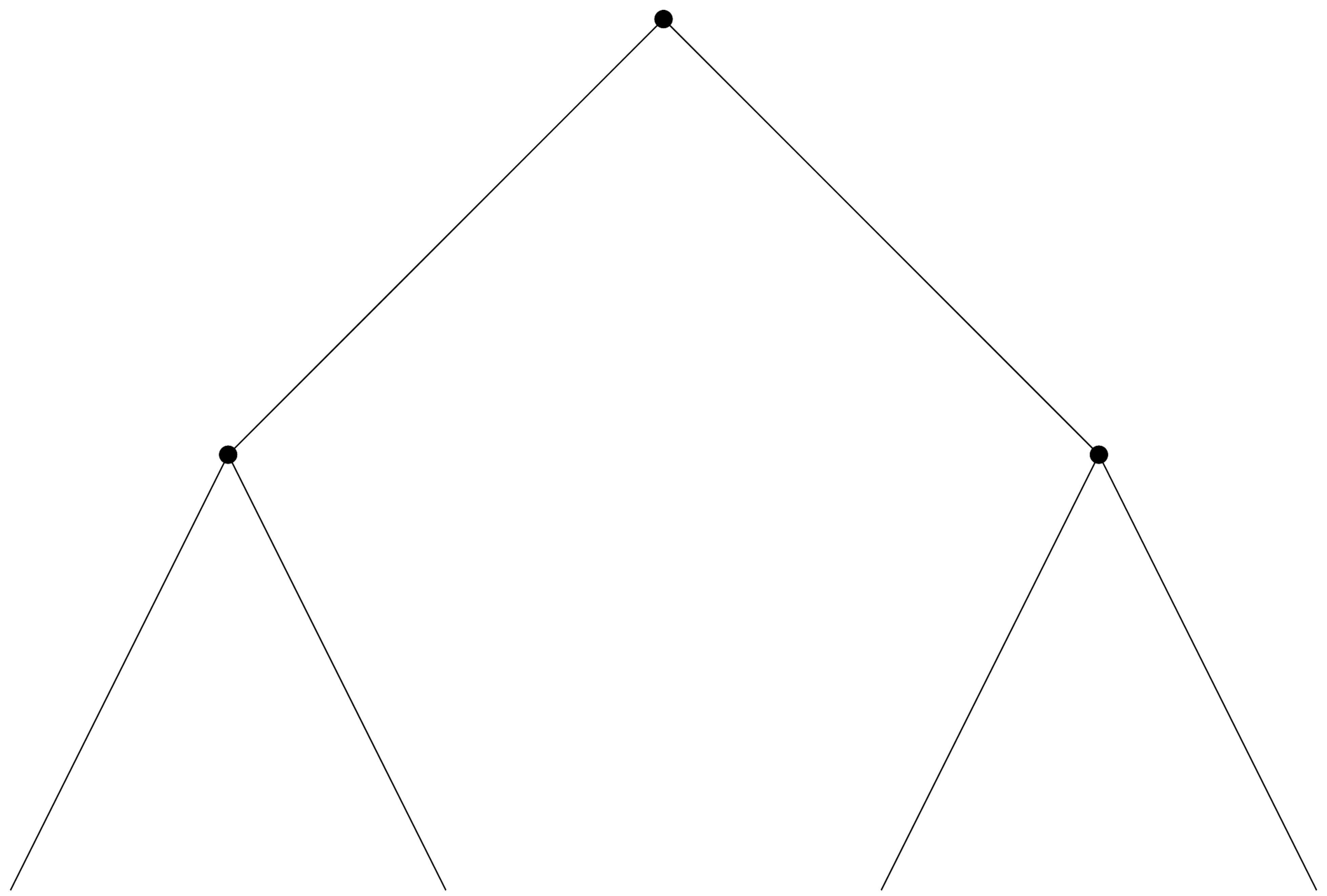
Game Tree



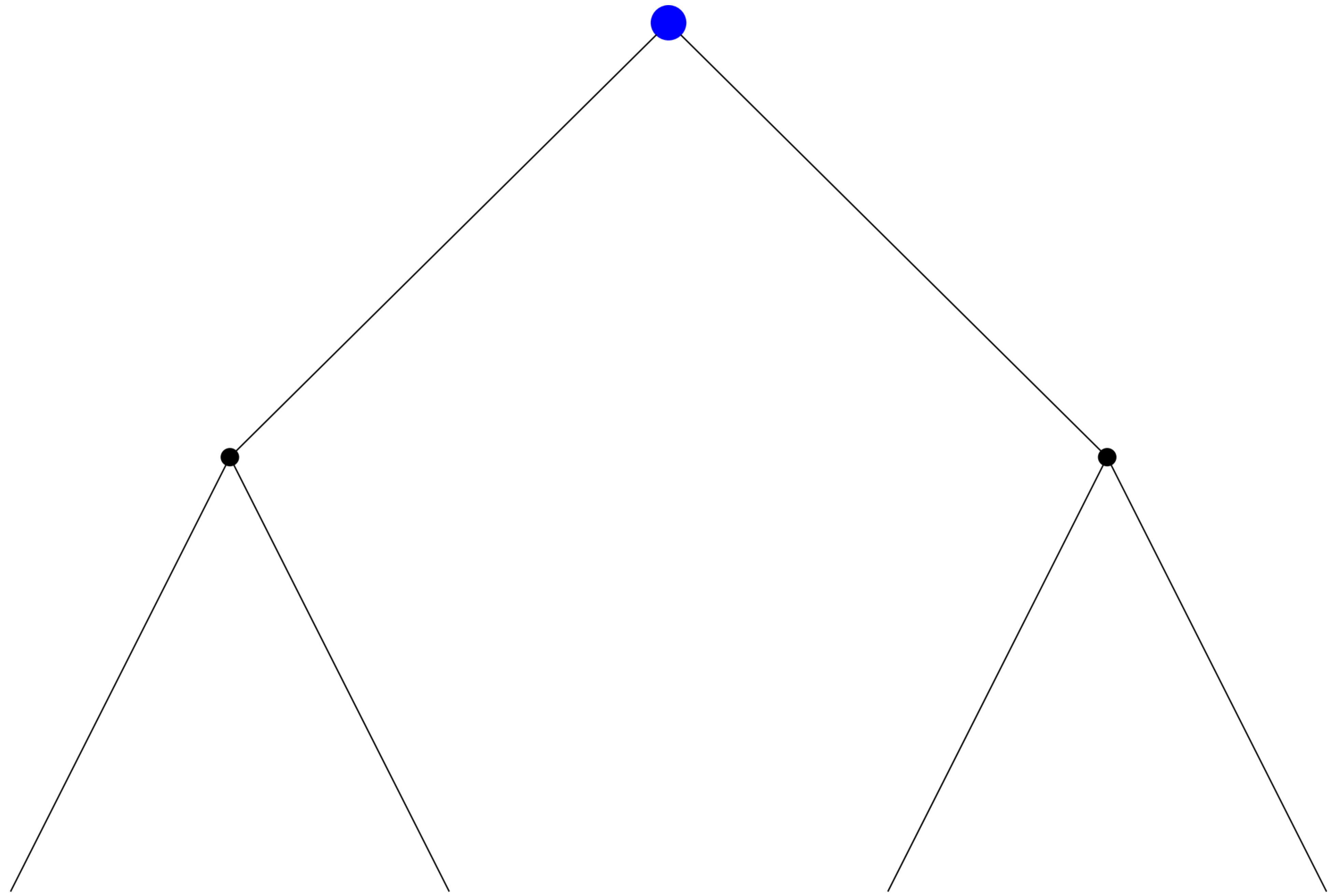
Player 1



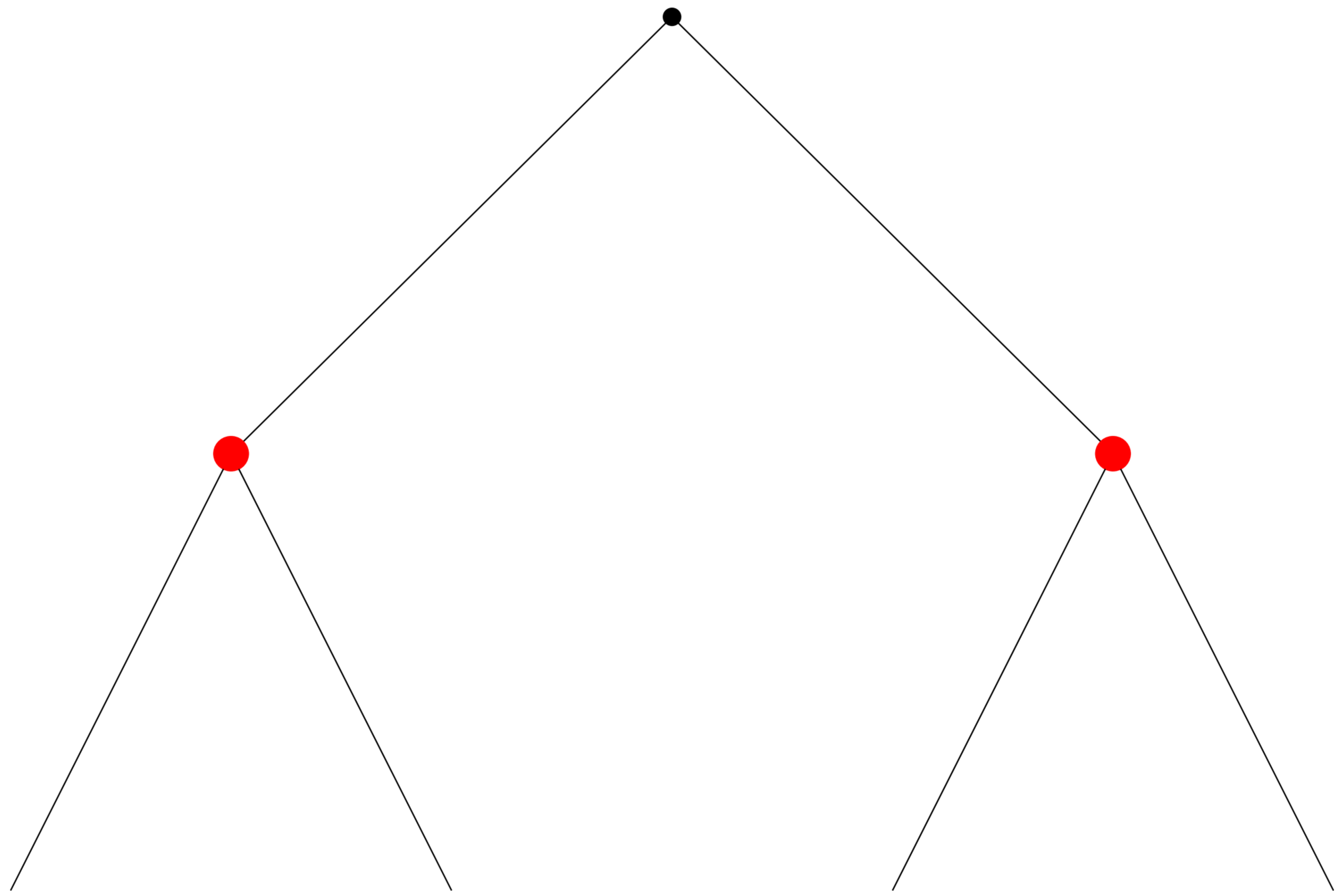
Player 2

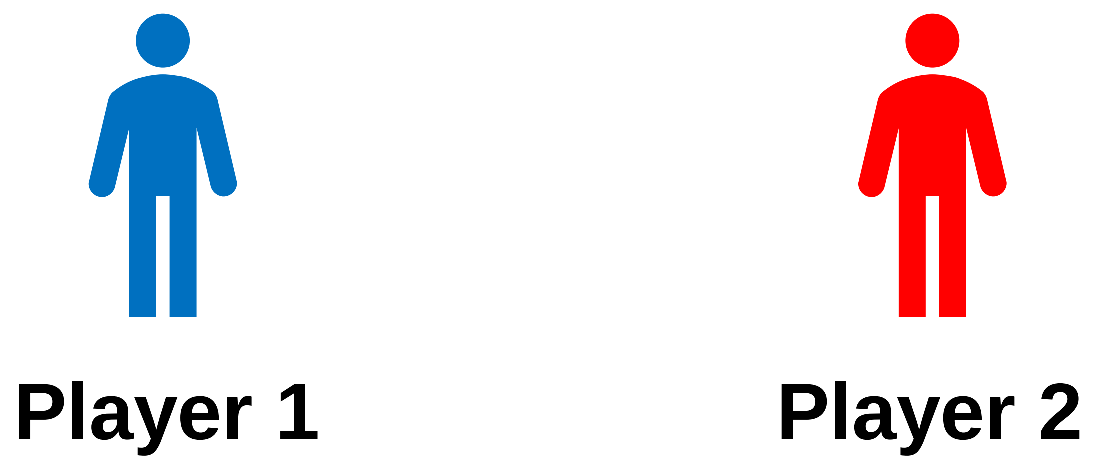


Game Tree

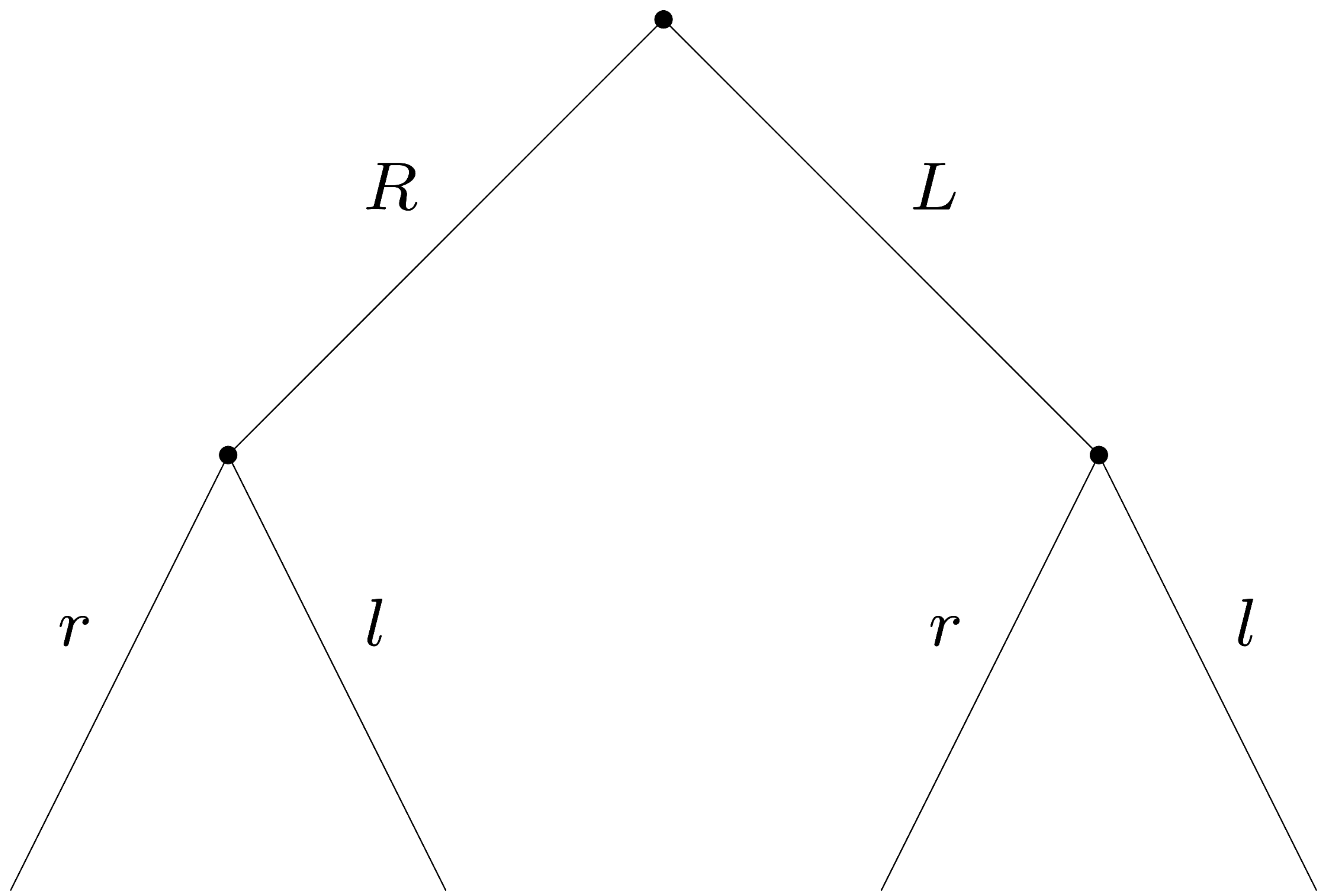


Game Tree

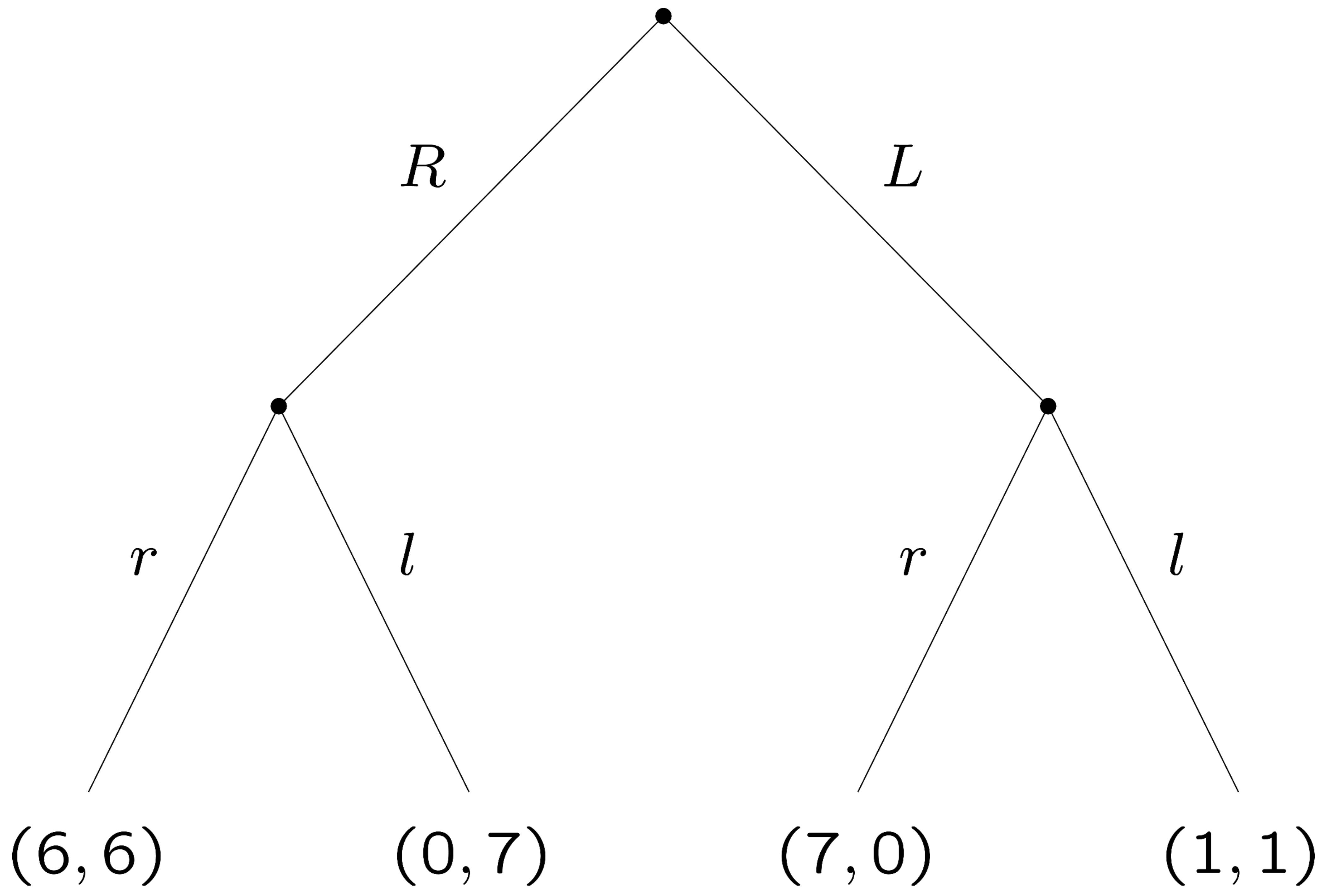




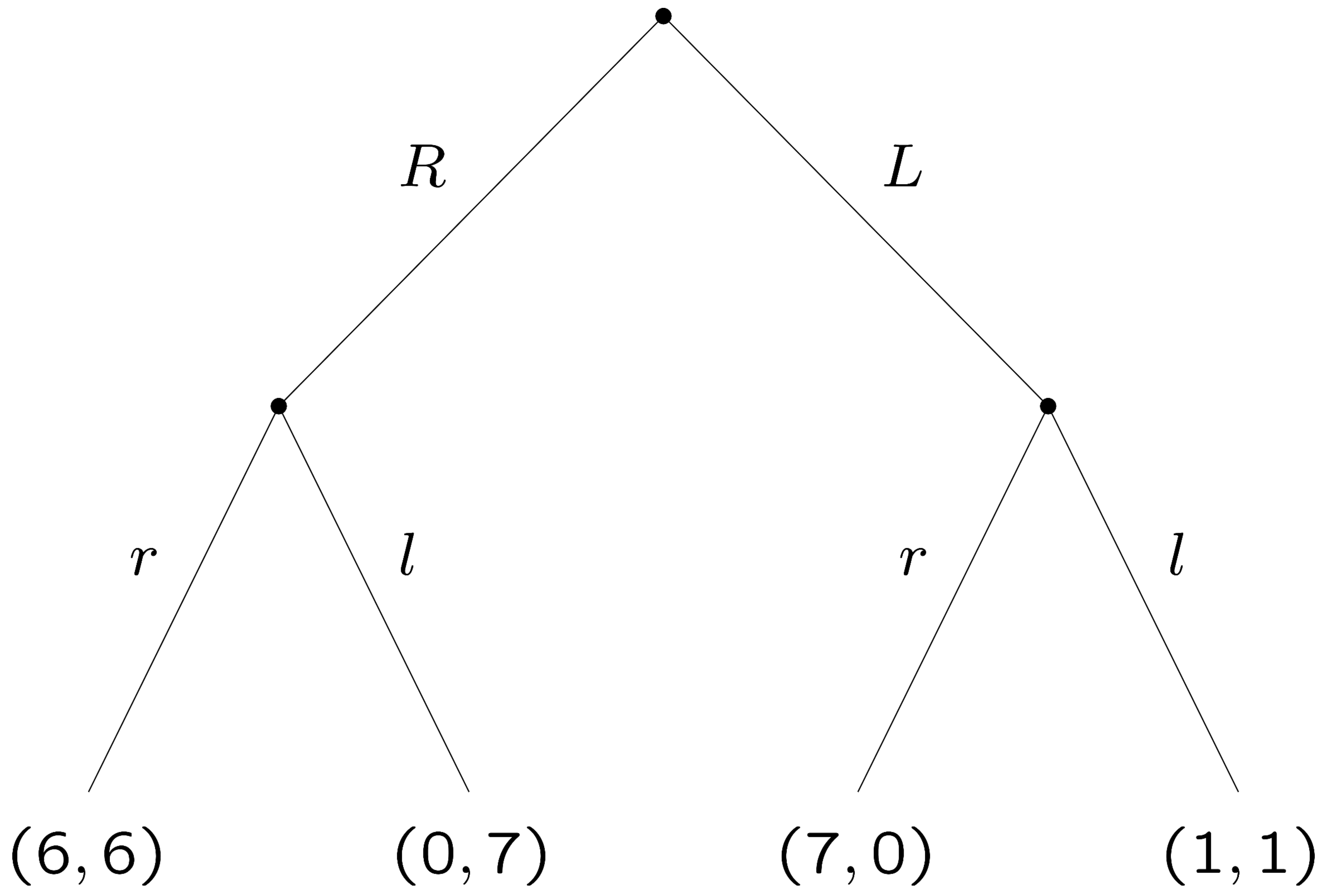
Game Tree



Game Tree

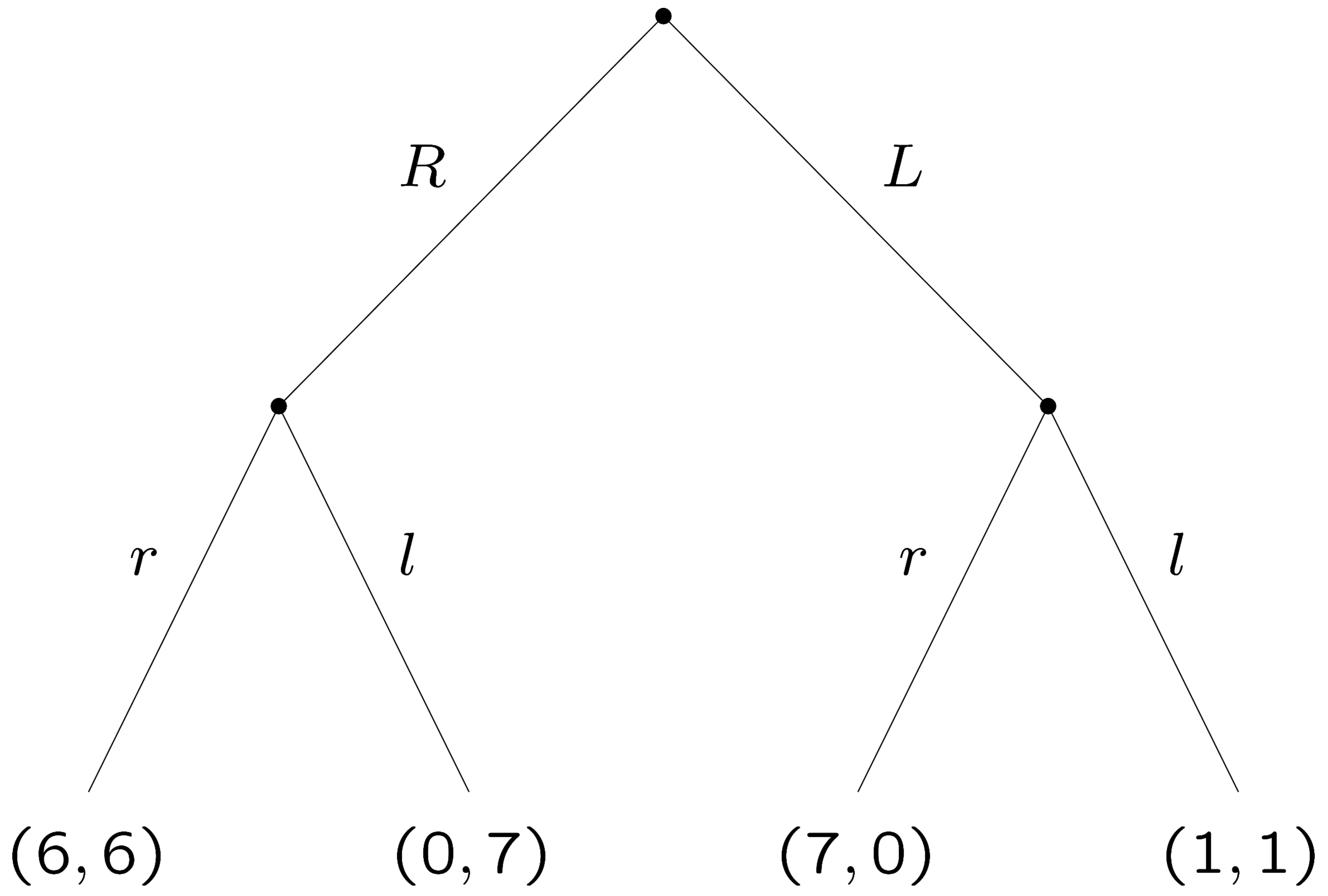


Game Tree



	r	***l***
R	(6,6)	(0,7)
L	(7,0)	(1,1)

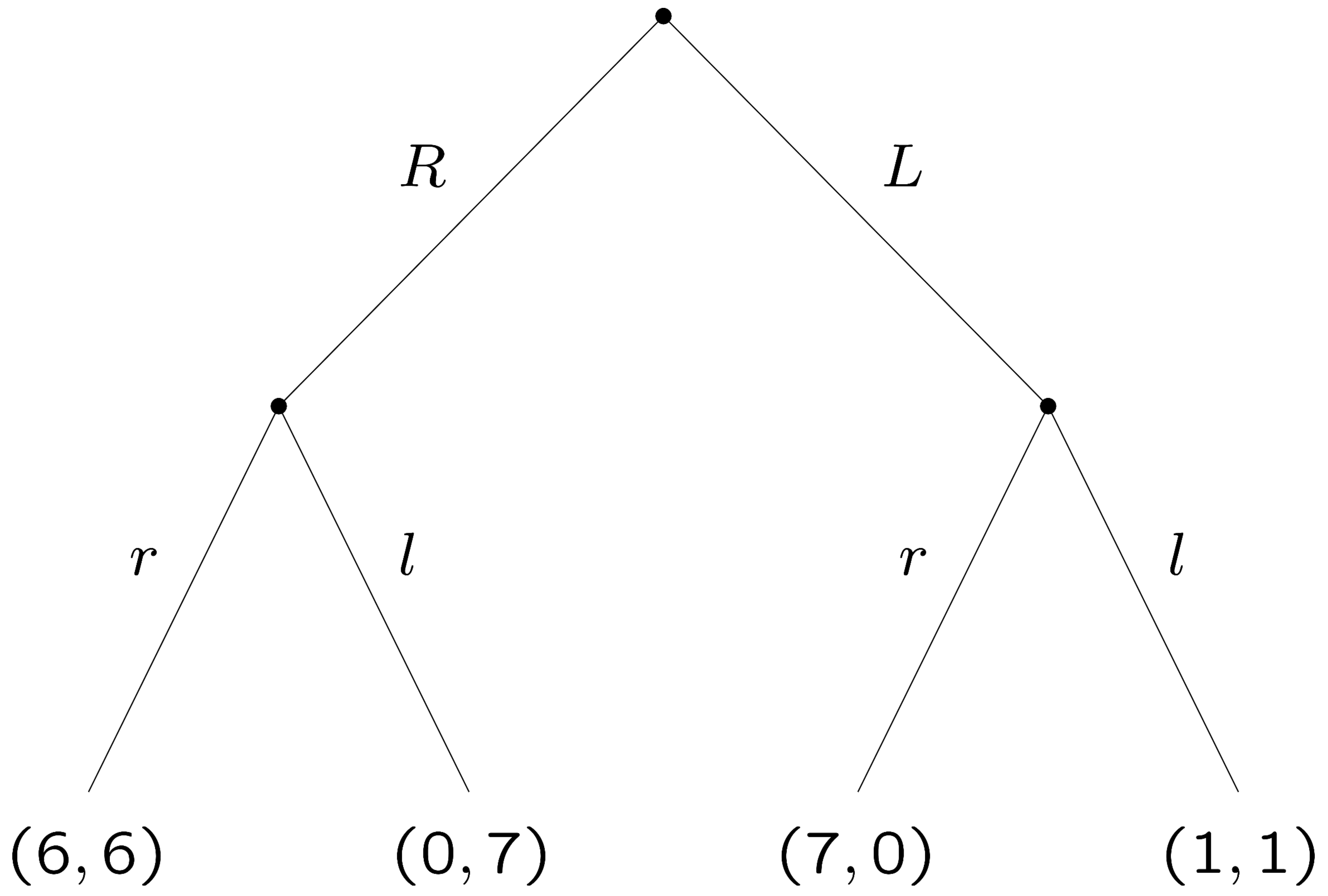
Game Tree



Player 1 **Player 2**

		r	***l***
0.75	***R***	(6,6)	(0,7)
0.25	***L***	(7,0)	(1,1)

Game Tree



Player 1 Player 2

		0.5	0.5
		r	***l***
0.75	***R***	(6,6)	(0,7)
0.25	***L***	(7,0)	(1,1)

Information and recall in games

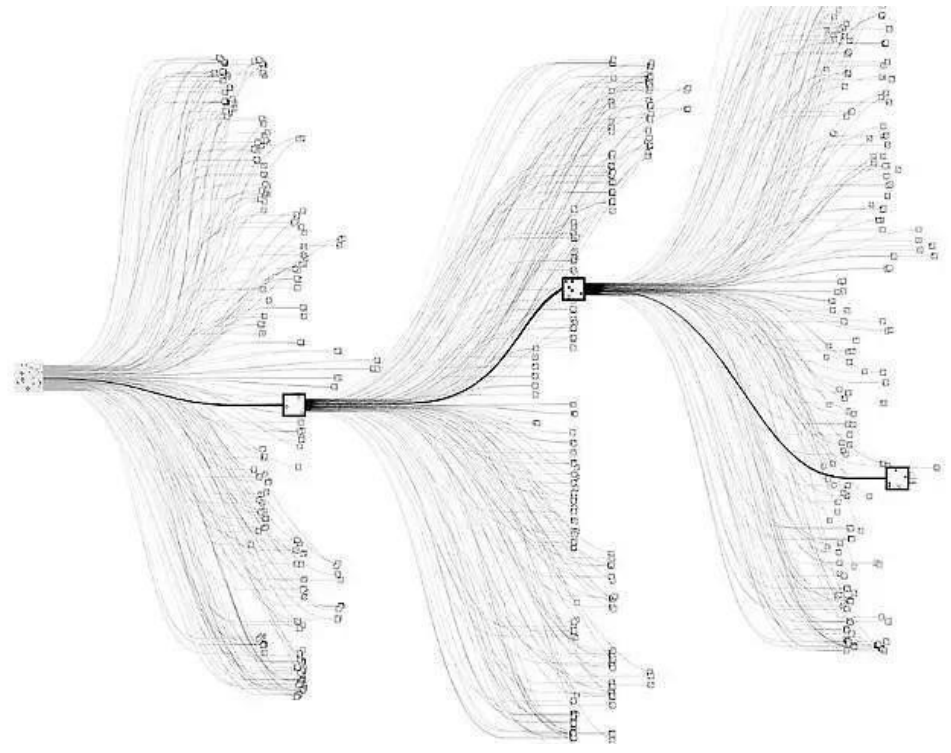
Perfect vs. imperfect information game

- In some games, defined as *perfect information* games, the state of the game is completely observable by the players

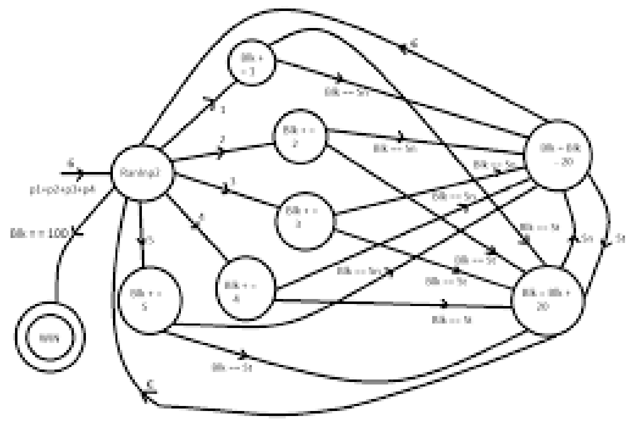
- When the state is not completely observable, the game is defined as *imperfect information* game

Perfect vs. imperfect recall game

- A *perfect recall* game is a game in which no player forgets information that he/she acquired before

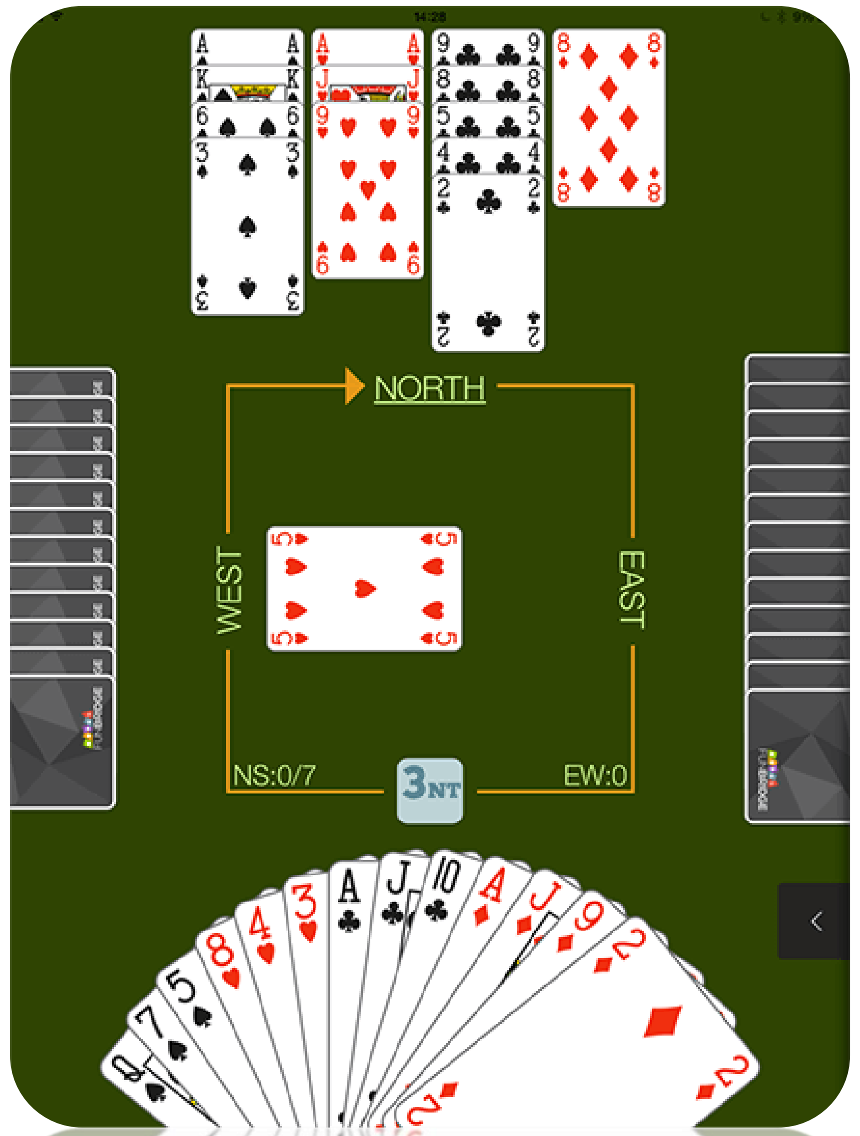


- An *imperfect recall* game is a game in which there is at least a player that is an imperfect recall player (e.g. it forgets some information that was known before in the game)



Team

- A team is a set of players that share the same objectives in the game



Bridge

Scopone Scientifico

- In Game Theory a team is modeled as a set of players that have the same utility function

Nash Equilibrium

- Solution concept introduced by John Nash in 1951

Nash Equilibrium

- Solution concept introduced by John Nash in 1951
- A NE is a joint combination of strategies stable with respect to unilateral deviations of a single player

Nash Equilibrium

- Solution concept introduced by John Nash in 1951

- A NE is a joint combination of strategies stable with respect to unilateral deviations of a single player

	r	l
R	(6,6)	(0,7)
L	(7,0)	(1,1)

Nash Equilibrium

- Solution concept introduced by John Nash in 1951
- A NE is a joint combination of strategies stable with respect to unilateral deviations of a single player

	r	l
R	(6,6)	(0,7)
L	(7,0)	(1,1)

Nash Equilibrium

- Solution concept introduced by John Nash in 1951

- A NE is a joint combination of strategies stable with respect to unilateral deviations of a single player

	r	l
R	(6,6)	(0,7)
L	(7,0)	(1,1)

- Approximation of Nash Equilibrium (ε-NE): joint combination of strategies such that no player can gain more that ε by unilaterally deviating

Team Maxmin Equilibrium

- Team Maxmin Equilibrium is the NE that maximizes the team utility
- From the team's perspective, a generic NE can be arbitrarily inefficient w.r.t. the TME

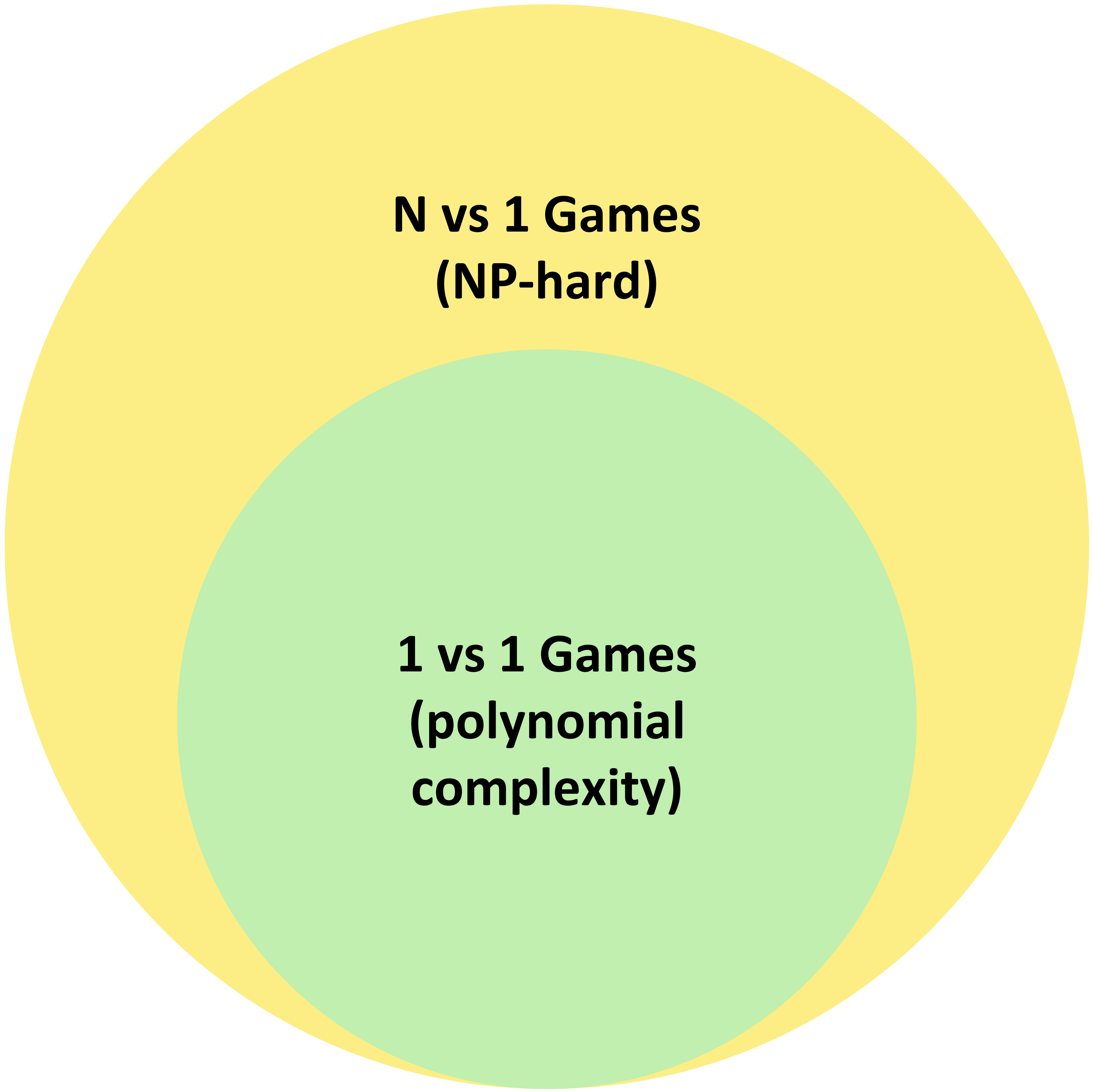
Outline

1. Introduction to Algorithmic Game Theory
2. Preliminaries
3. **State of the art**
4. Project proposal

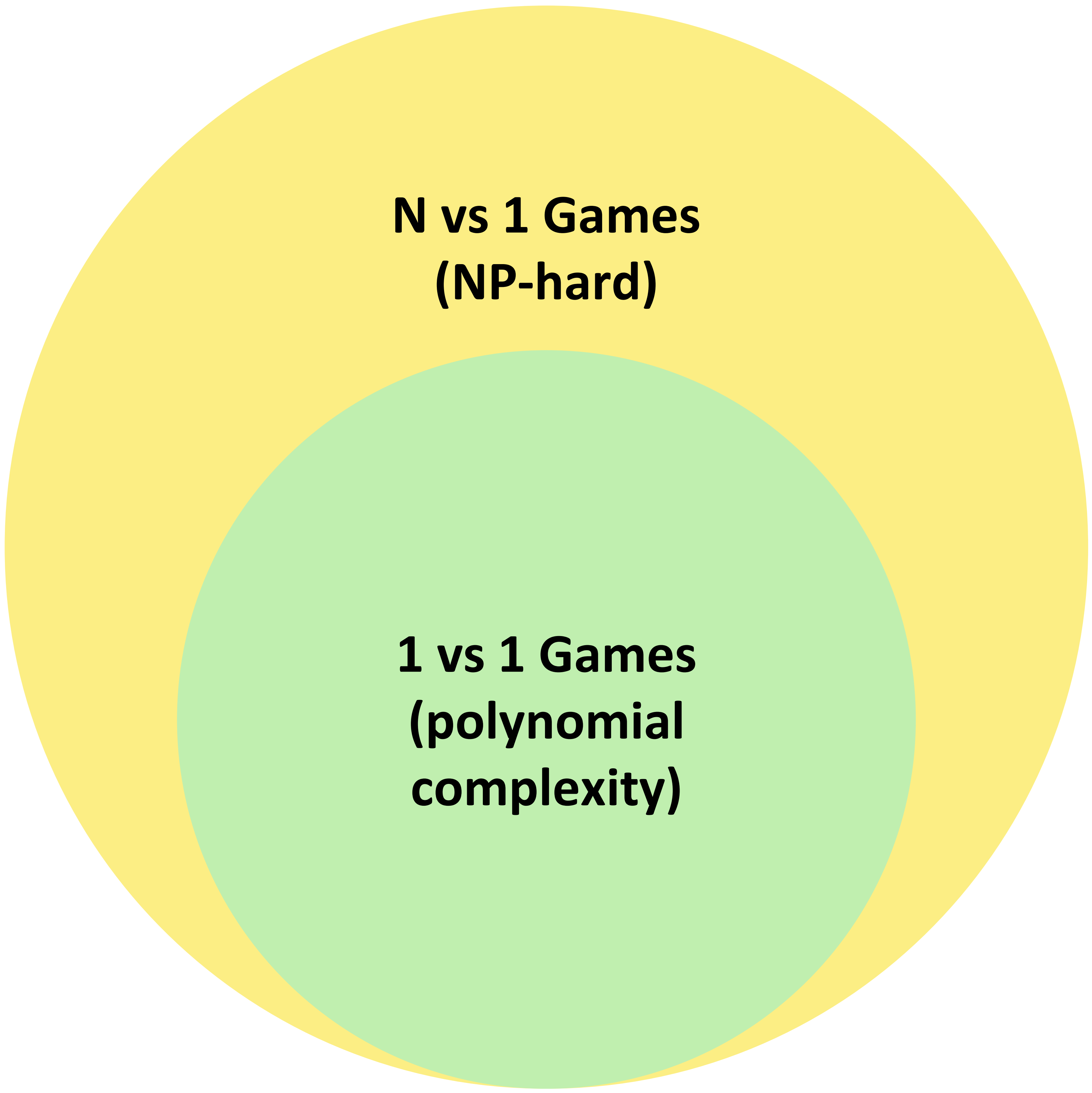
State of the art

1 vs 1 Games
(polynomial
complexity)

State of the art



State of the art



Introduce state-of-the-art algorithms for two-player games

↓

Explain source of NP-hardness of solving team games

↓

Introduce state-of-the-art algorithms for team games

State of the art: 1 vs. 1 games

Fictitious Play: (Slow convergence rate)

$$\epsilon \sim O(\frac{1}{\sqrt[n]{T}})$$

- Fictitious Play (FP) (Brown, 1951)
  - Weakened FP (van der Genugten, 2000)
  - Fictitious Self Play (Heinrich et al., 2015)
  - Neural FSP (Brown et al., 2018)

No-regret learning: (Fast convergence rate)

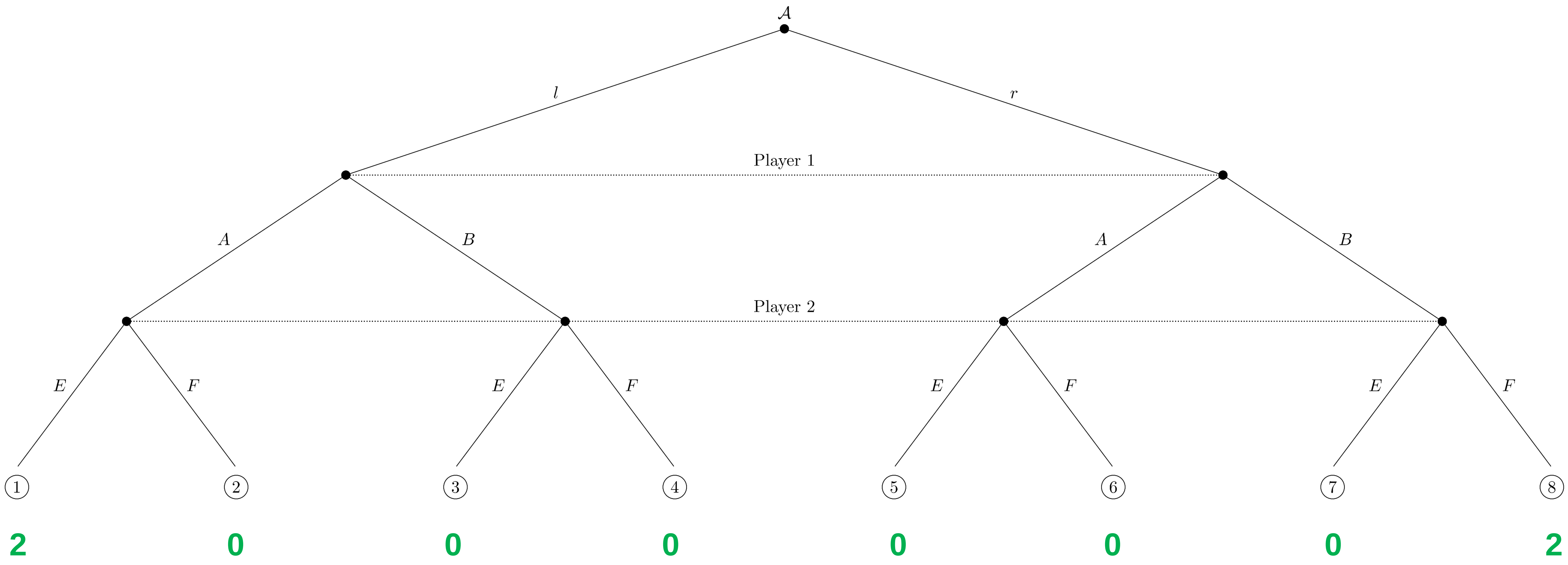
$$\epsilon \sim O(\frac{1}{\sqrt{T}})$$

- Counterfactual Regret Minimization (CFR) (Zinkevich et al., 2008)
  - Monte Carlo CFR (Lanctot et al., 2009)
  - CFR-BR (Johanson et al., 2012)
  - Deep-CFR (Brown et al., 2018)

State of the Art: N vs. 1

- From the perspective of a team, not correlating the strategies of the teammates can be inefficient, in a measure depending on the number of players and on the number of available actions (Basilico et al., 2016)

- Focus on 2 vs 1 games



State of the Art: N vs. 1

State of the Art: N vs. 1

Two different models of communication between team members, supported by different devices:

State of the Art: N vs. 1

Two different models of communication between team members, supported by different devices:

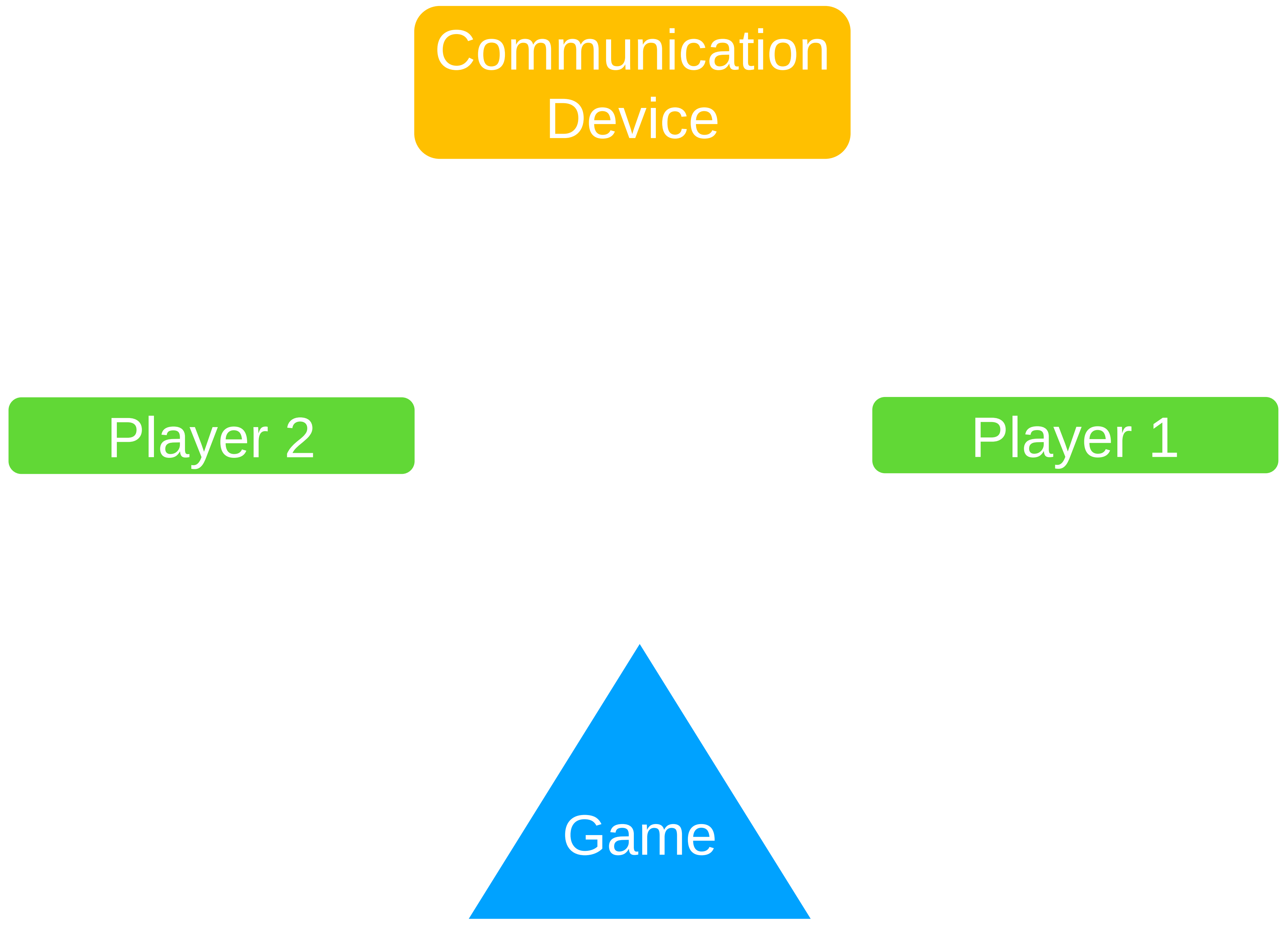
Communication Device:

State of the Art: N vs. 1

Two different models of communication between team members, supported by different devices:

Communication Device:

- *Preplay* and *Intraplay* communication

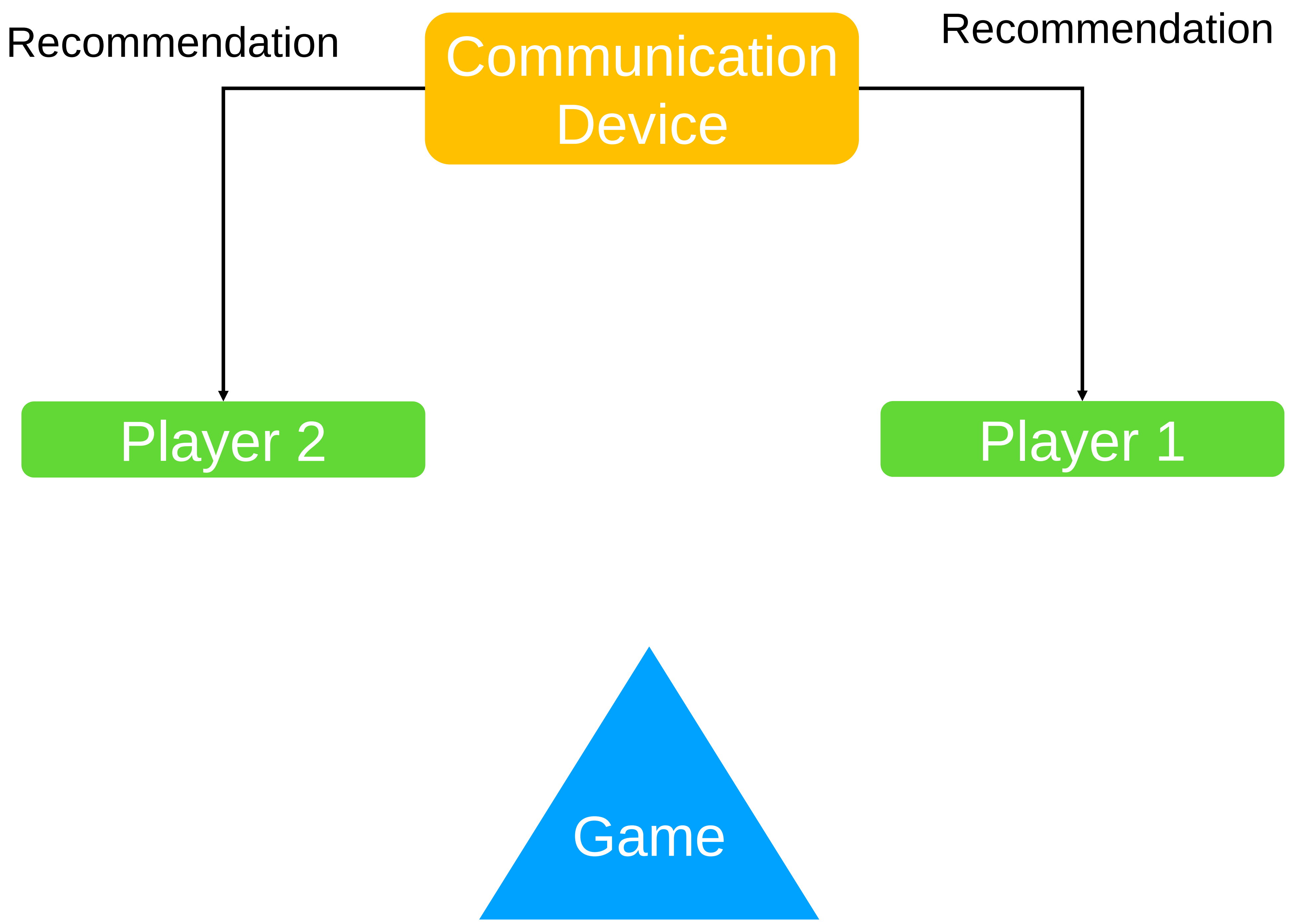


State of the Art: N vs. 1

Two different models of communication between team members, supported by different devices:

Communication Device:

- *Preplay* and *Intraplay* communication

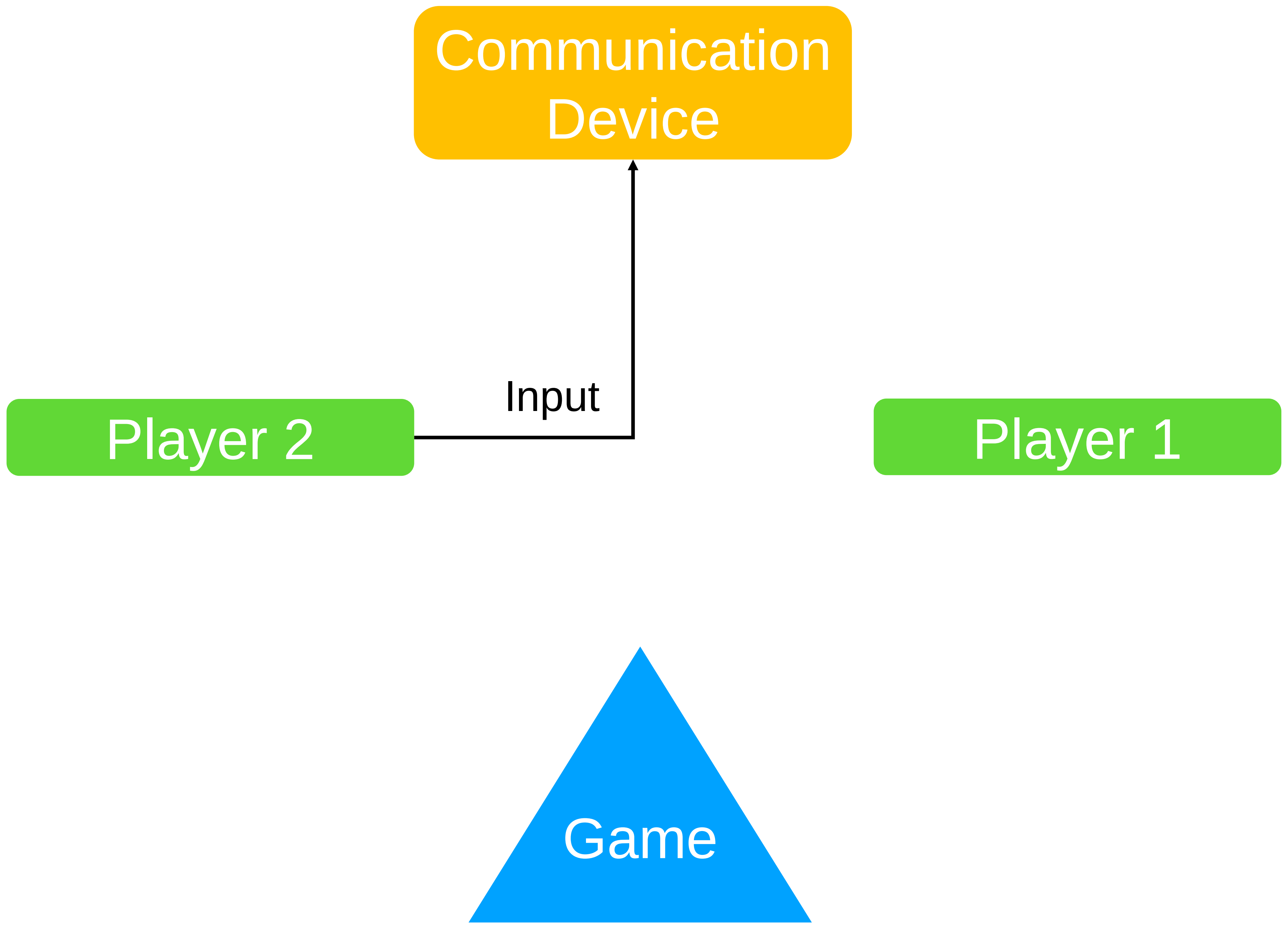


State of the Art: N vs. 1

Two different models of communication between team members, supported by different devices:

Communication Device:

- *Preplay* and *Intraplay* communication

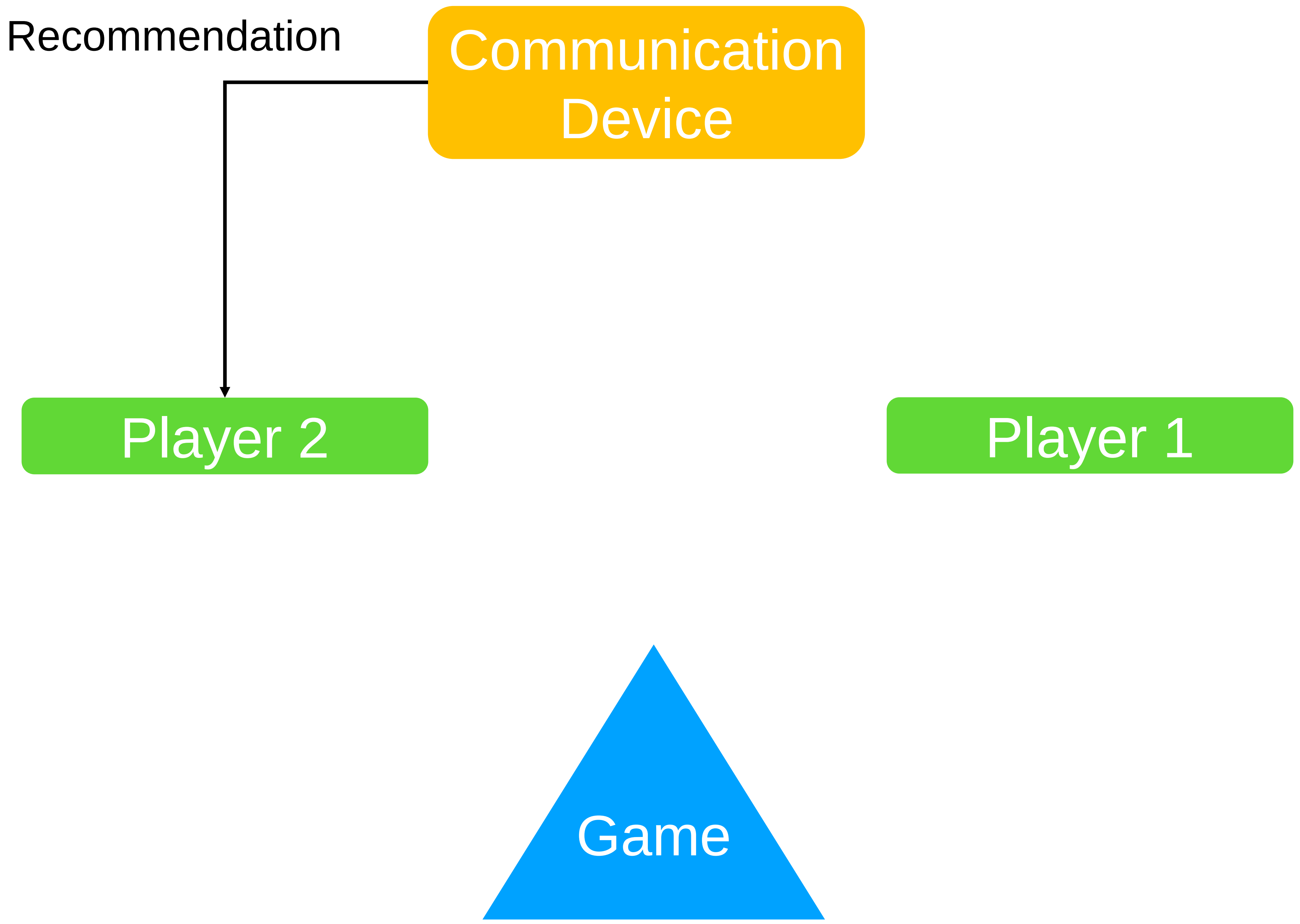


State of the Art: N vs. 1

Two different models of communication between team members, supported by different devices:

Communication Device:

- *Preplay* and *Intraplay* communication

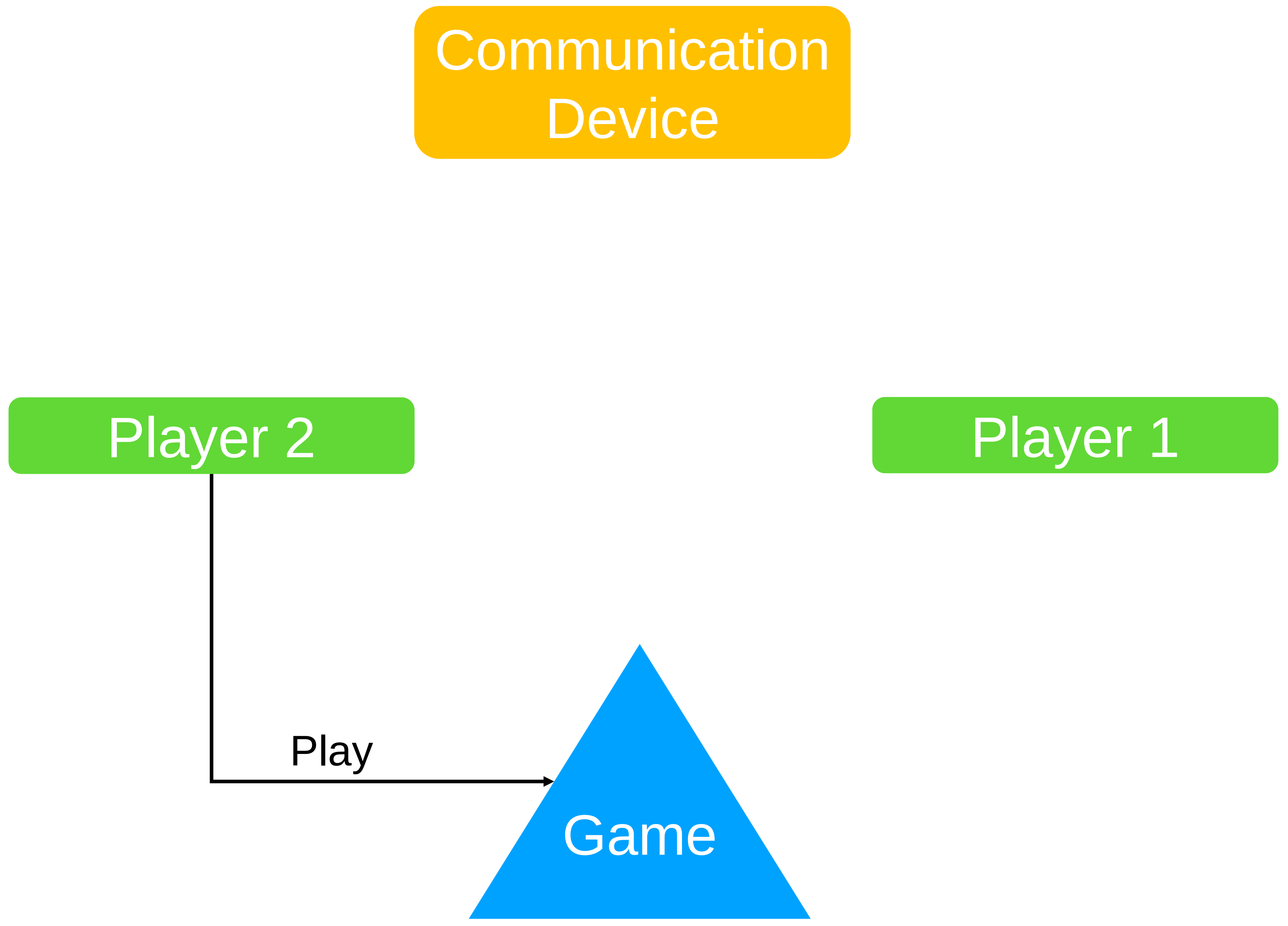


State of the Art: N vs. 1

Two different models of communication between team members, supported by different devices:

Communication Device:

- *Preplay* and *Intraplay* communication

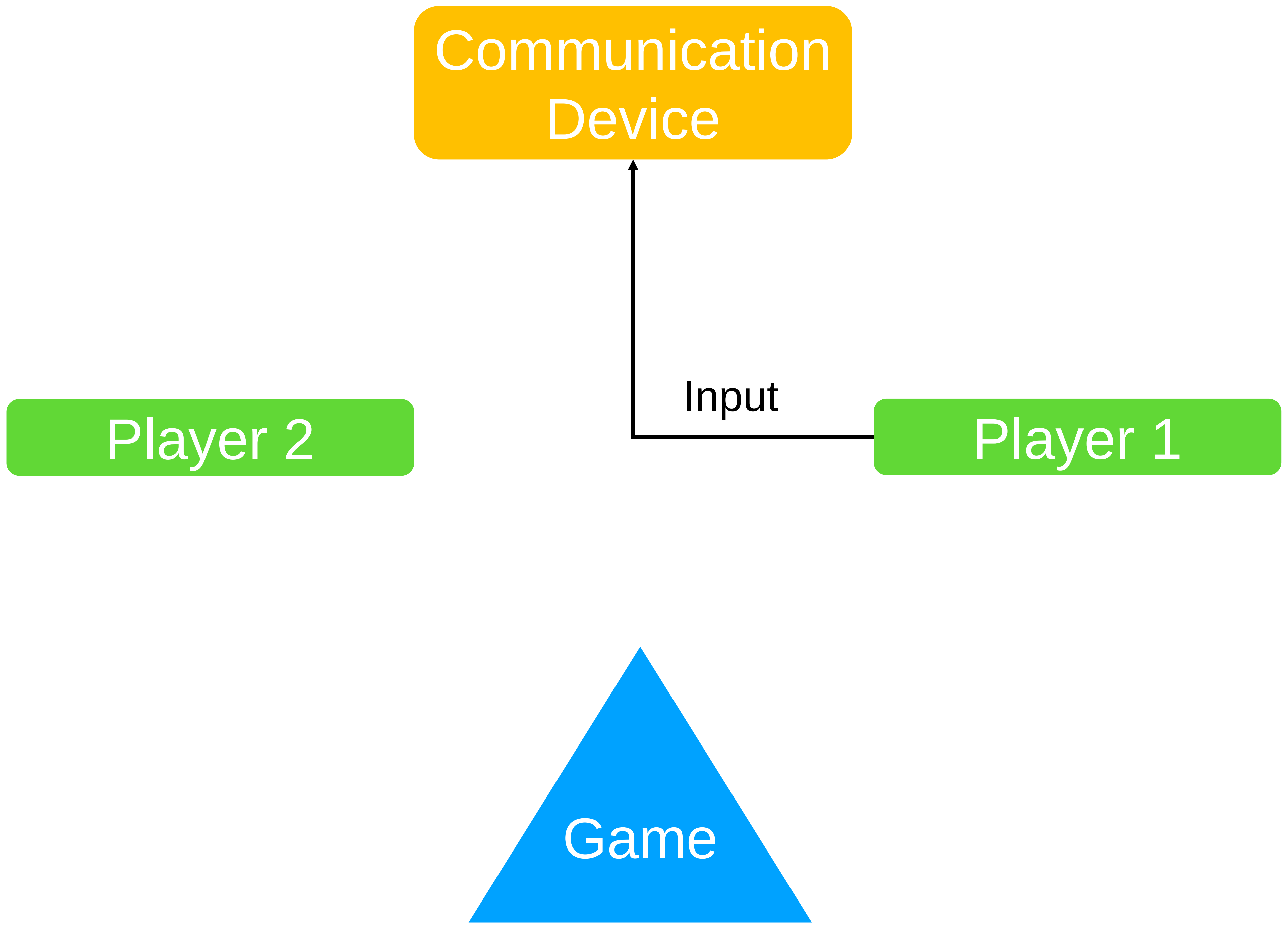


State of the Art: N vs. 1

Two different models of communication between team members, supported by different devices:

Communication Device:

- *Preplay* and *Intraplay* communication

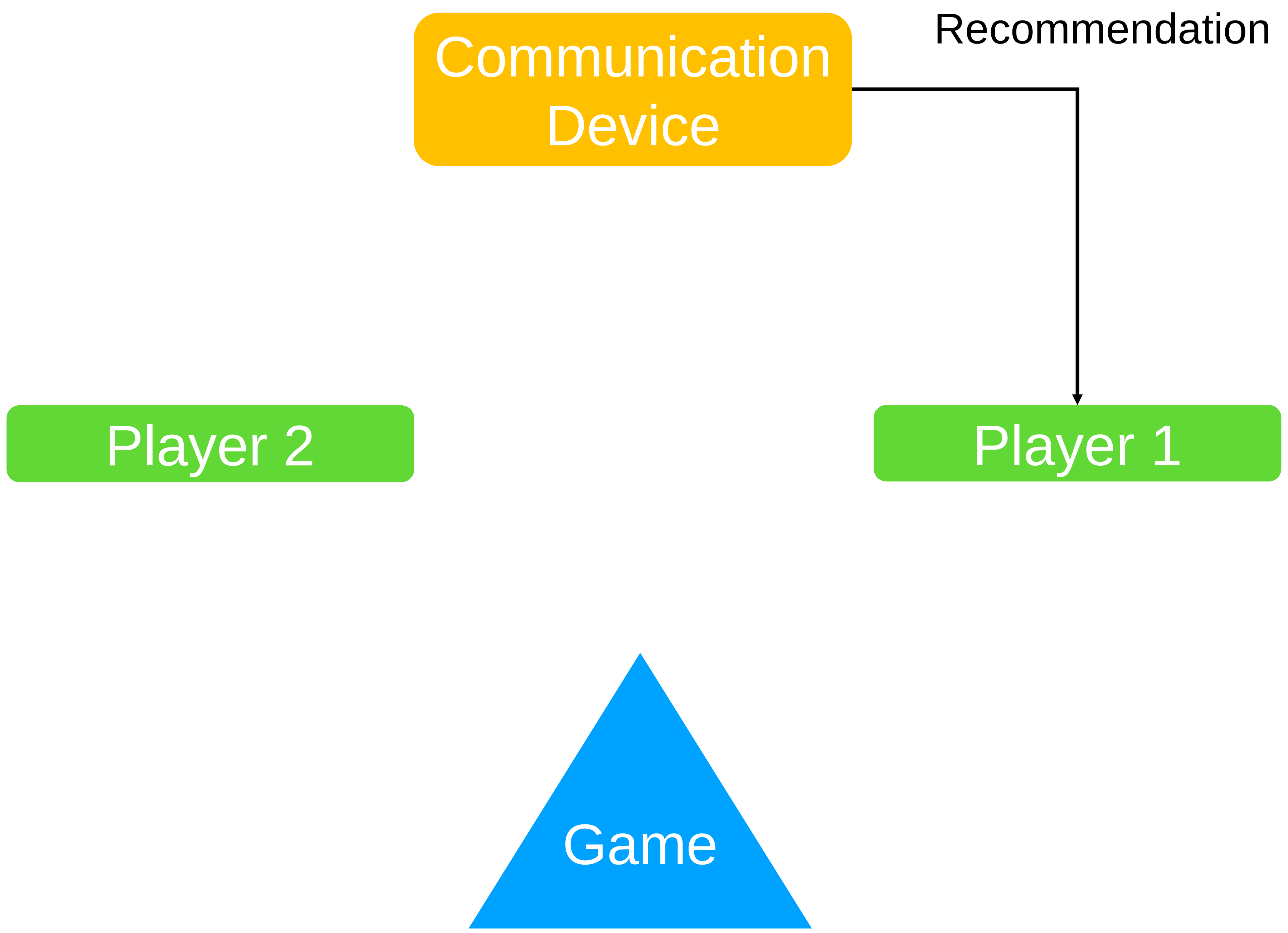


State of the Art: N vs. 1

Two different models of communication between team members, supported by different devices:

Communication Device:

- *Preplay* and *Intraplay* communication

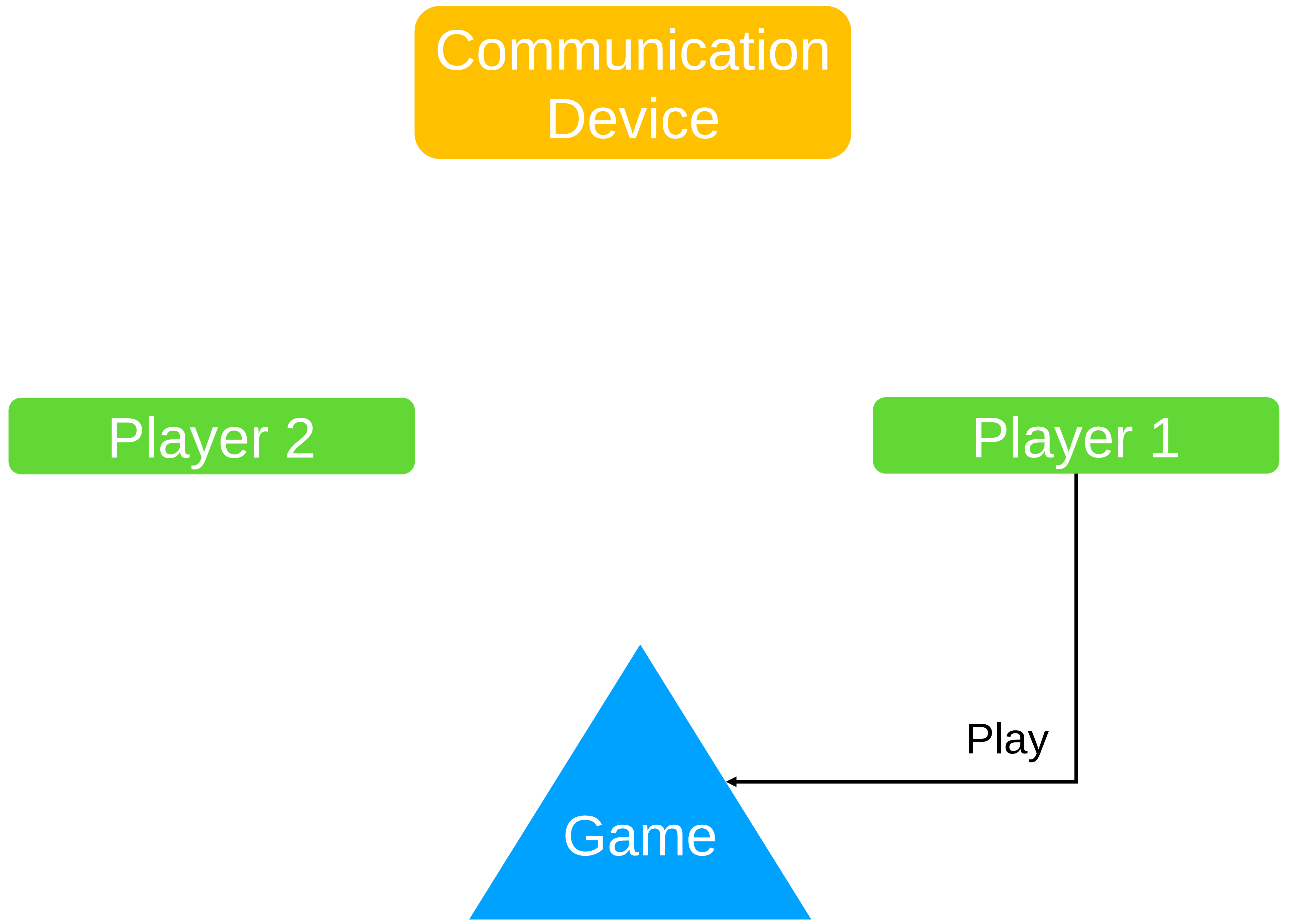


State of the Art: N vs. 1

Two different models of communication between team members, supported by different devices:

Communication Device:

- *Preplay* and *Intraplay* communication

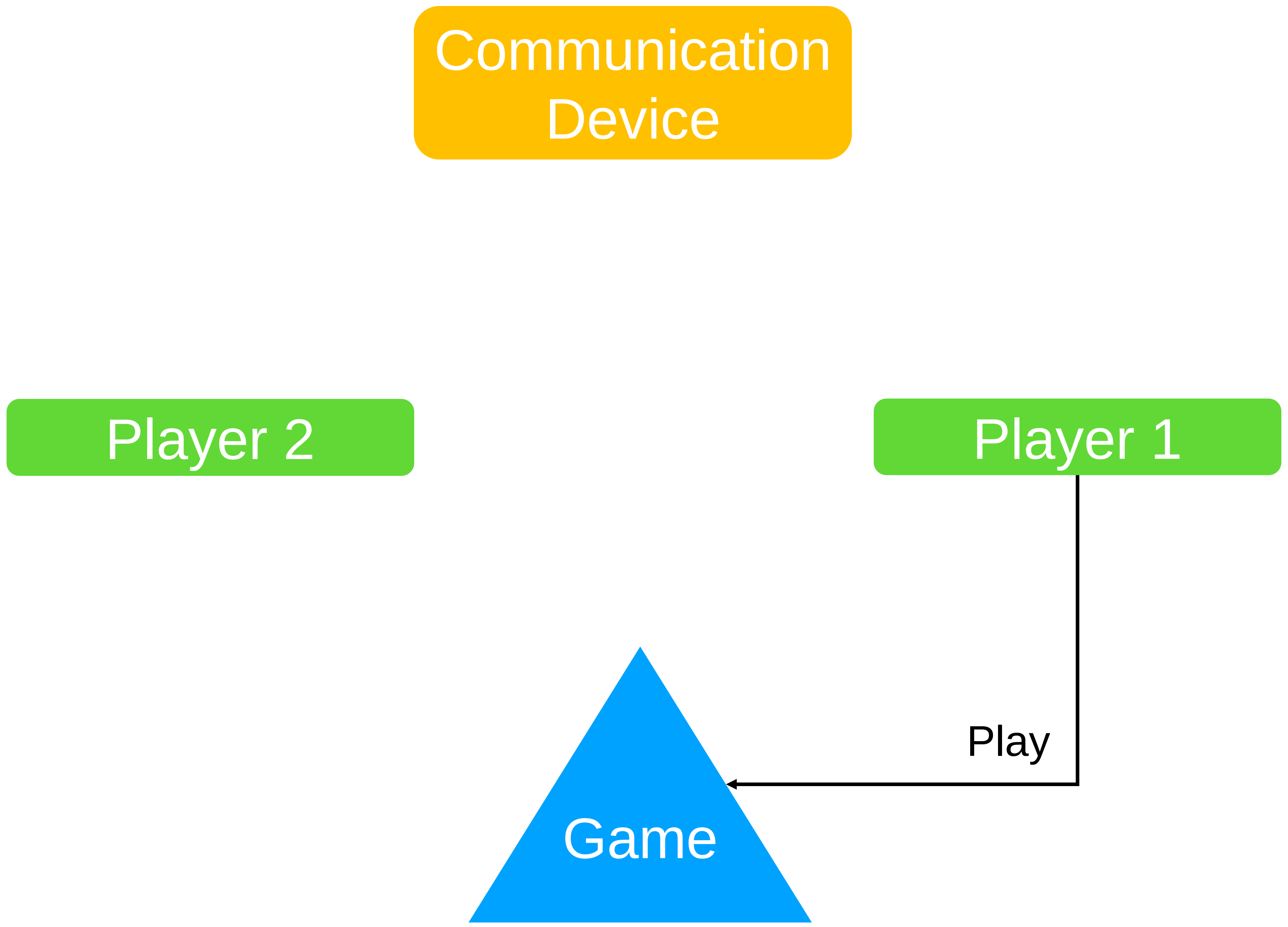


State of the Art: N vs. 1

Two different models of communication between team members, supported by different devices:

Communication Device:

- *Preplay* and *Intraplay* communication
- *TMEcom*

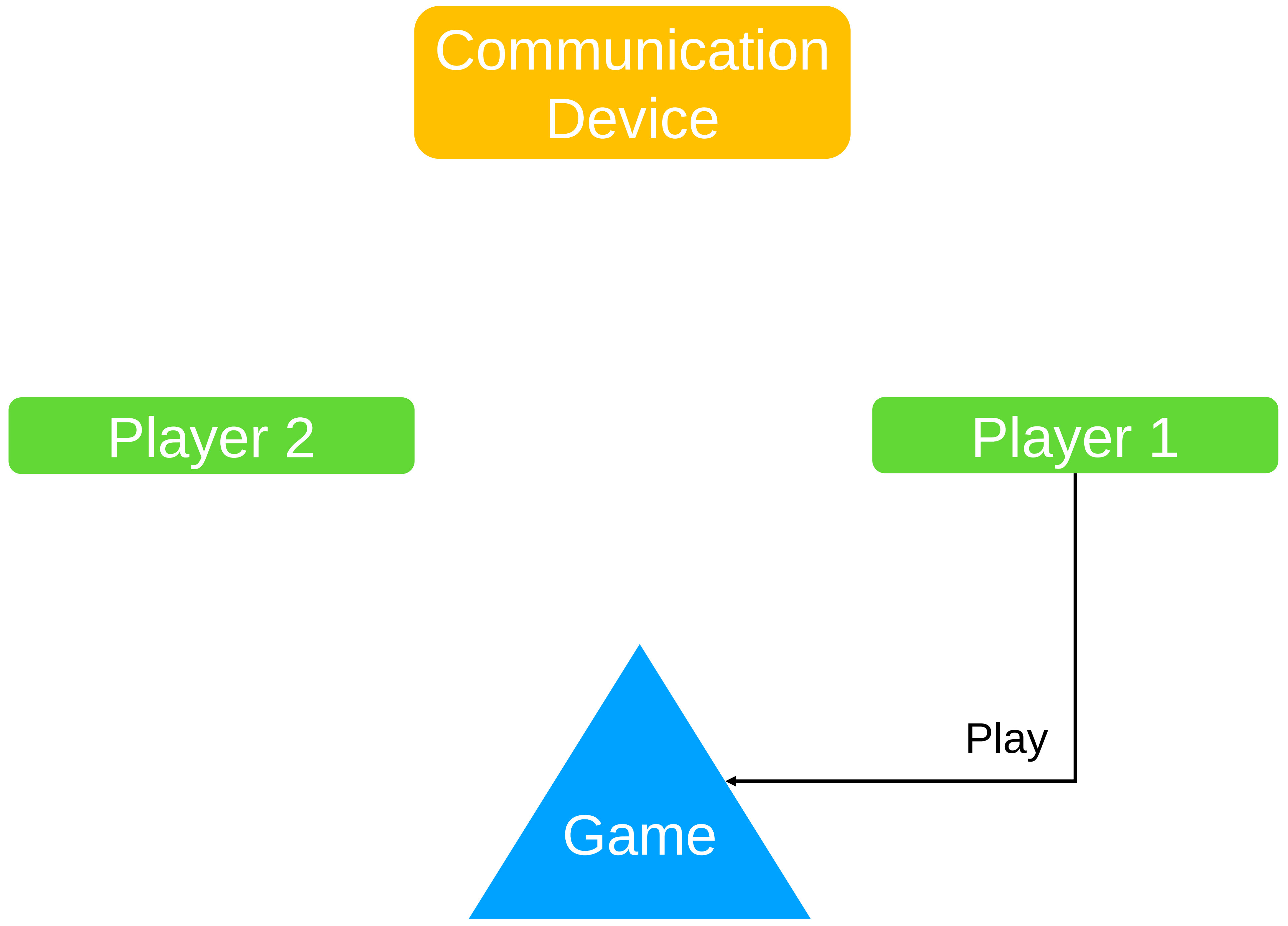


State of the Art: N vs. 1

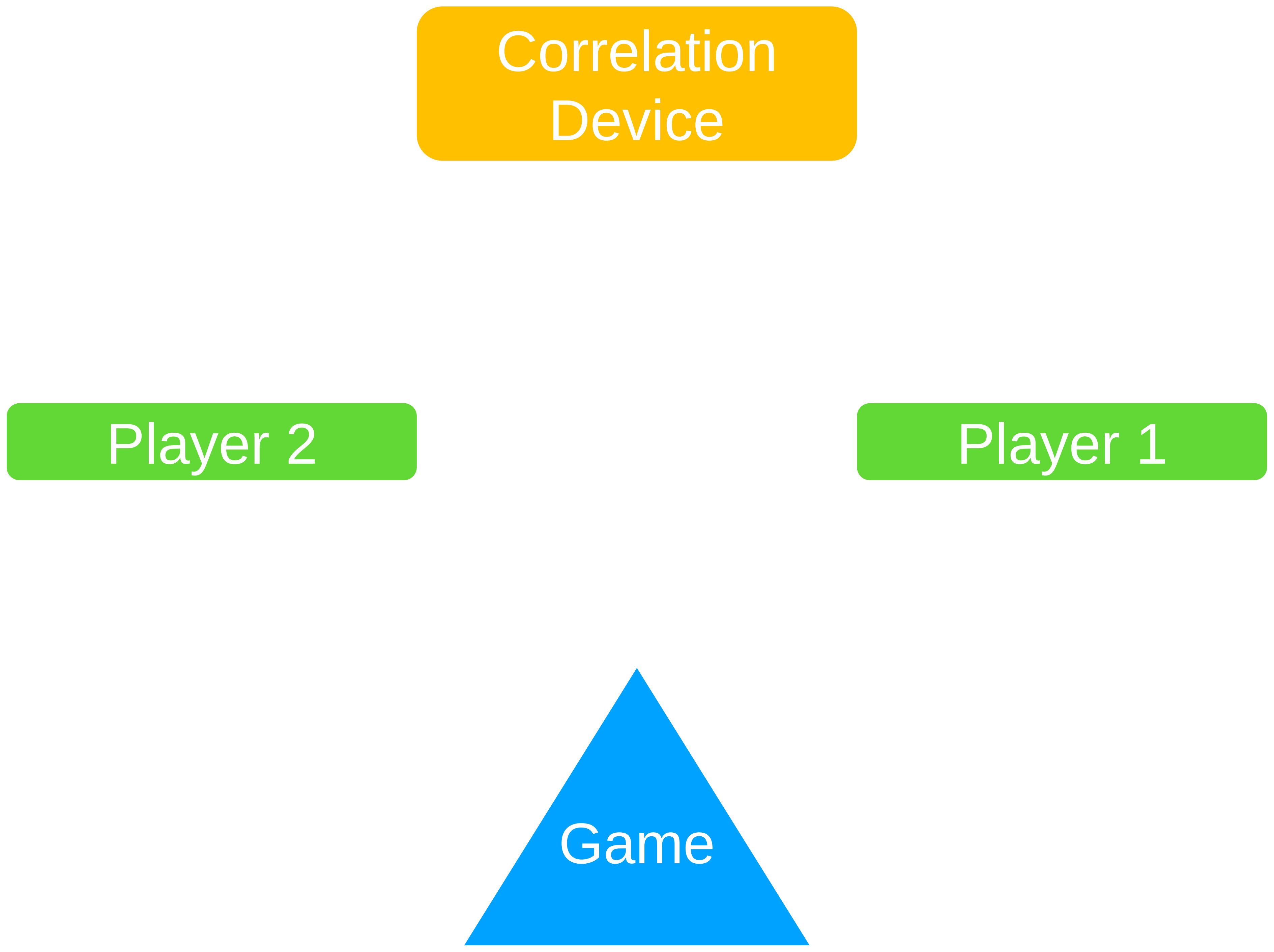
Two different models of communication between team members, supported by different devices:

Communication Device:

- *Preplay* and *Intraplay* communication
- *TMEcom*



Correlation Device:

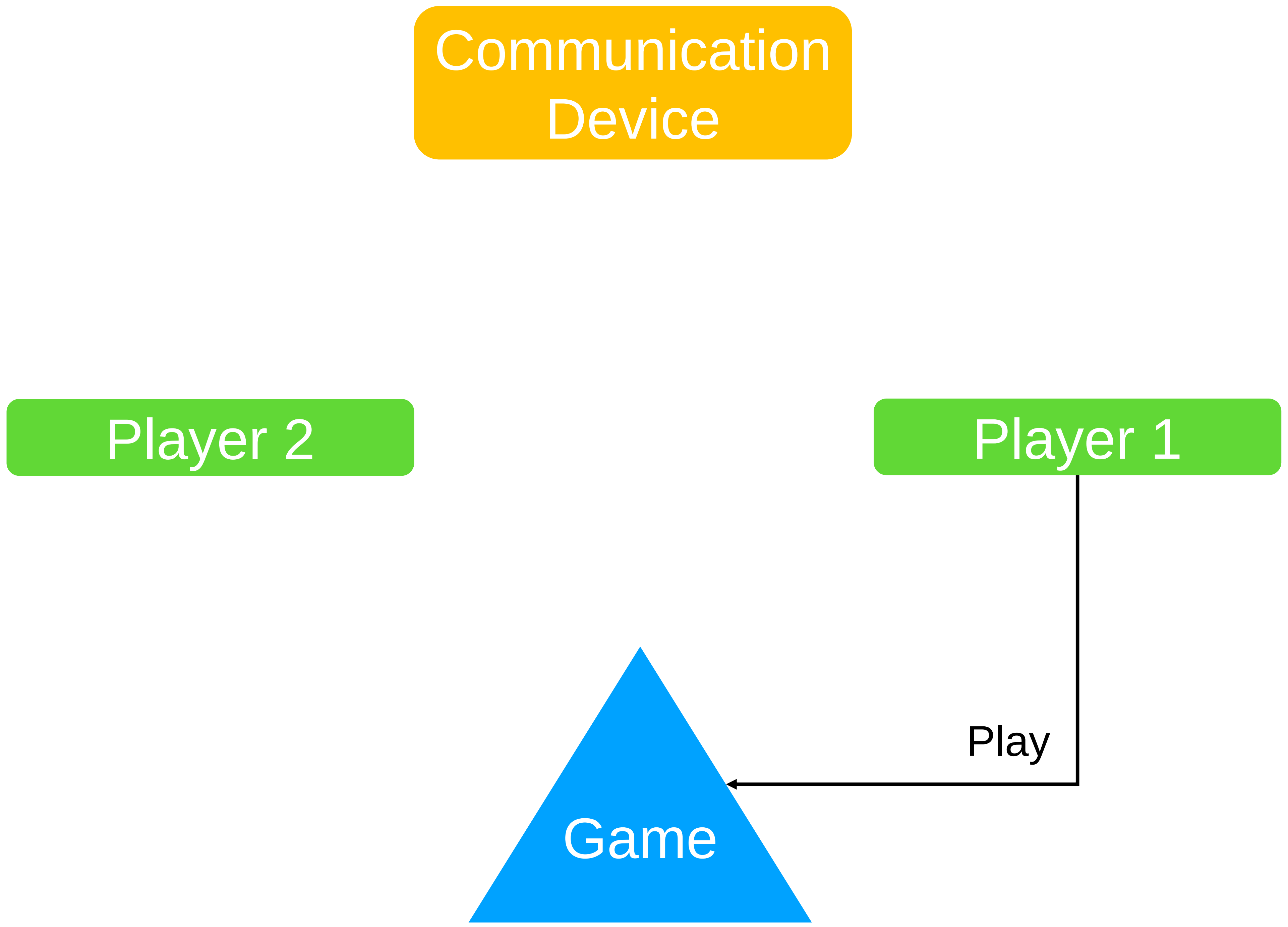


State of the Art: N vs. 1

Two different models of communication between team members, supported by different devices:

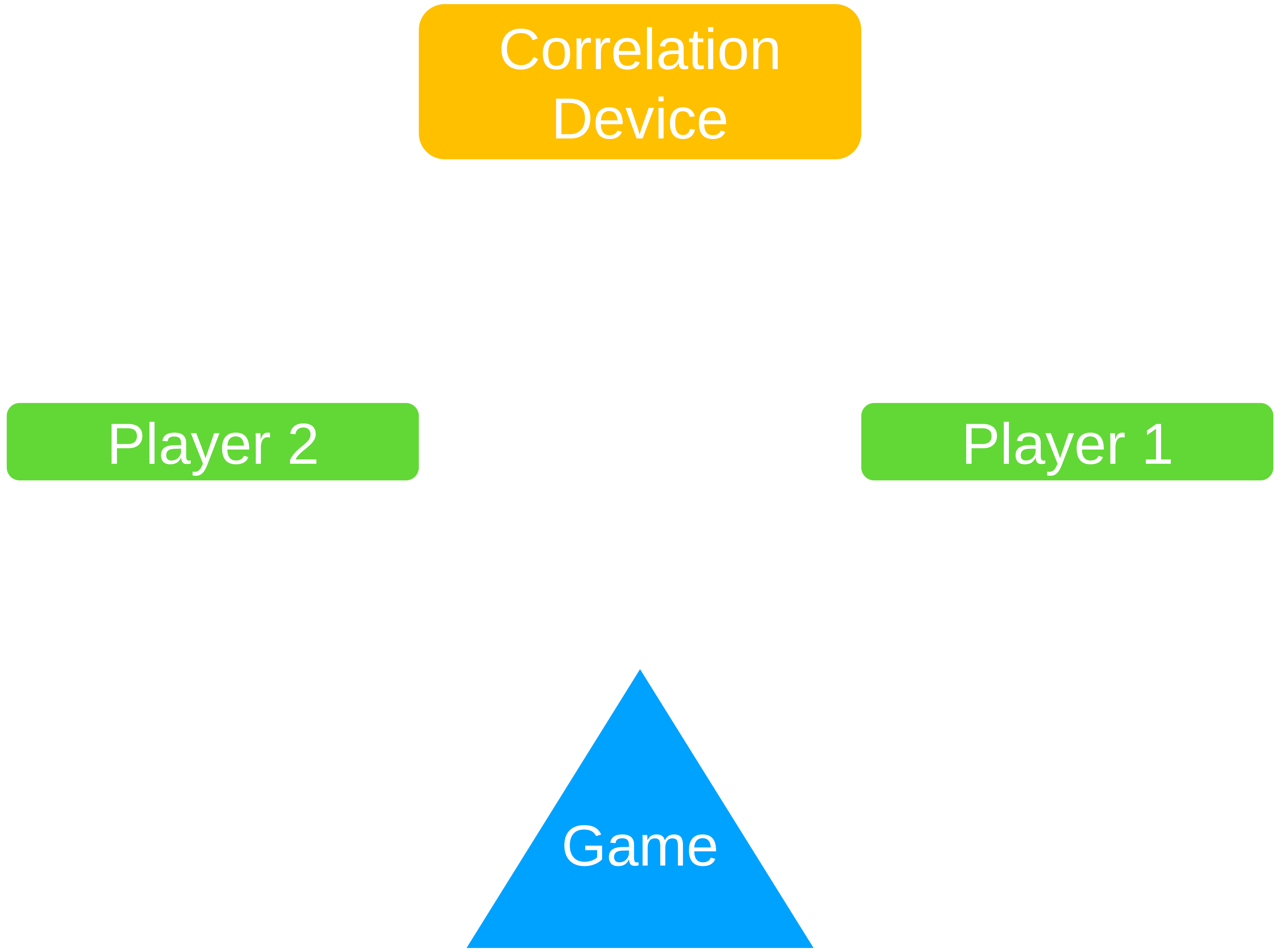
Communication Device:

- *Preplay* and *Intraplay* communication
- *TMEcom*



Correlation Device:

- *Preplay* communication

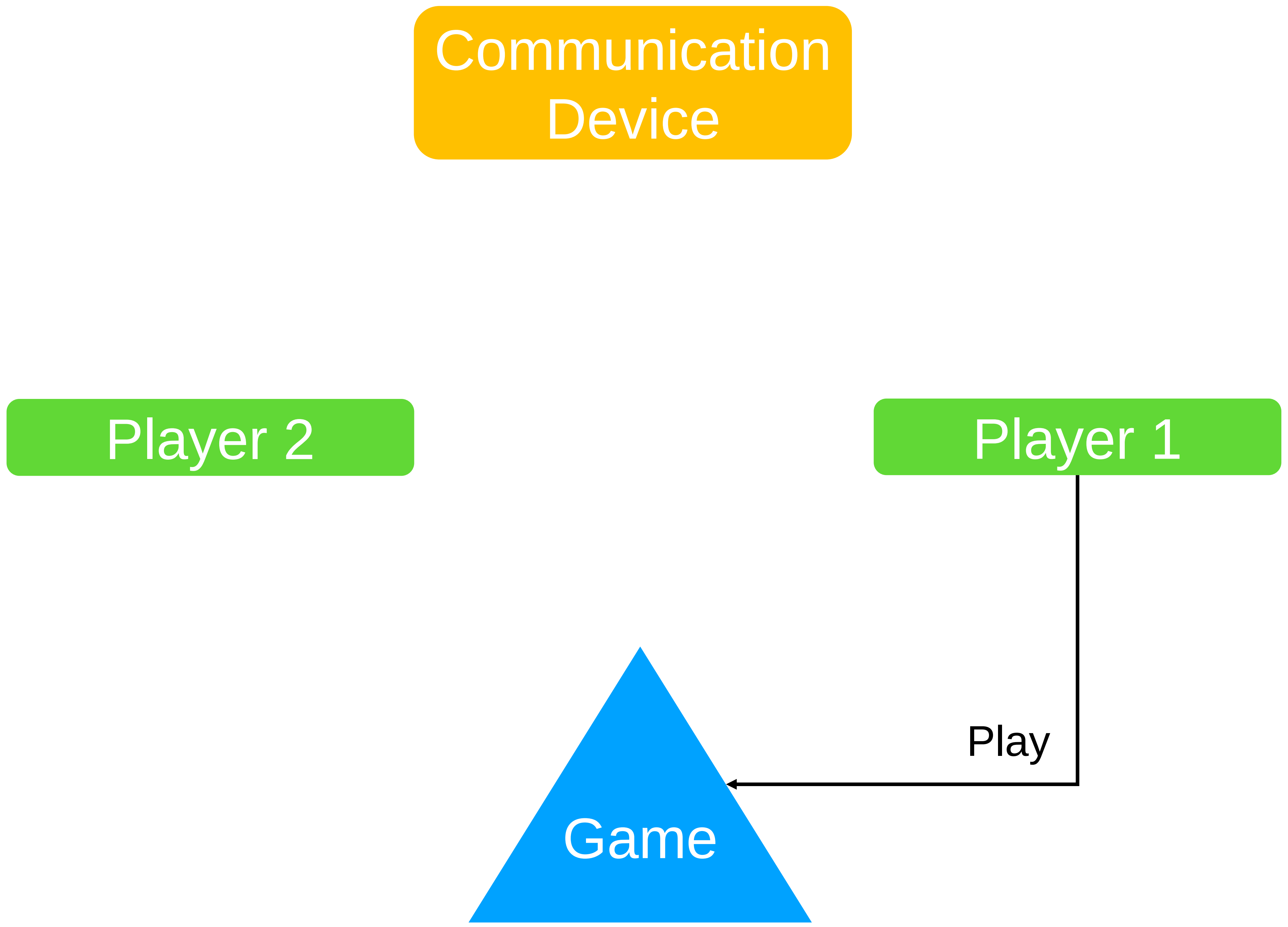


State of the Art: N vs. 1

Two different models of communication between team members, supported by different devices:

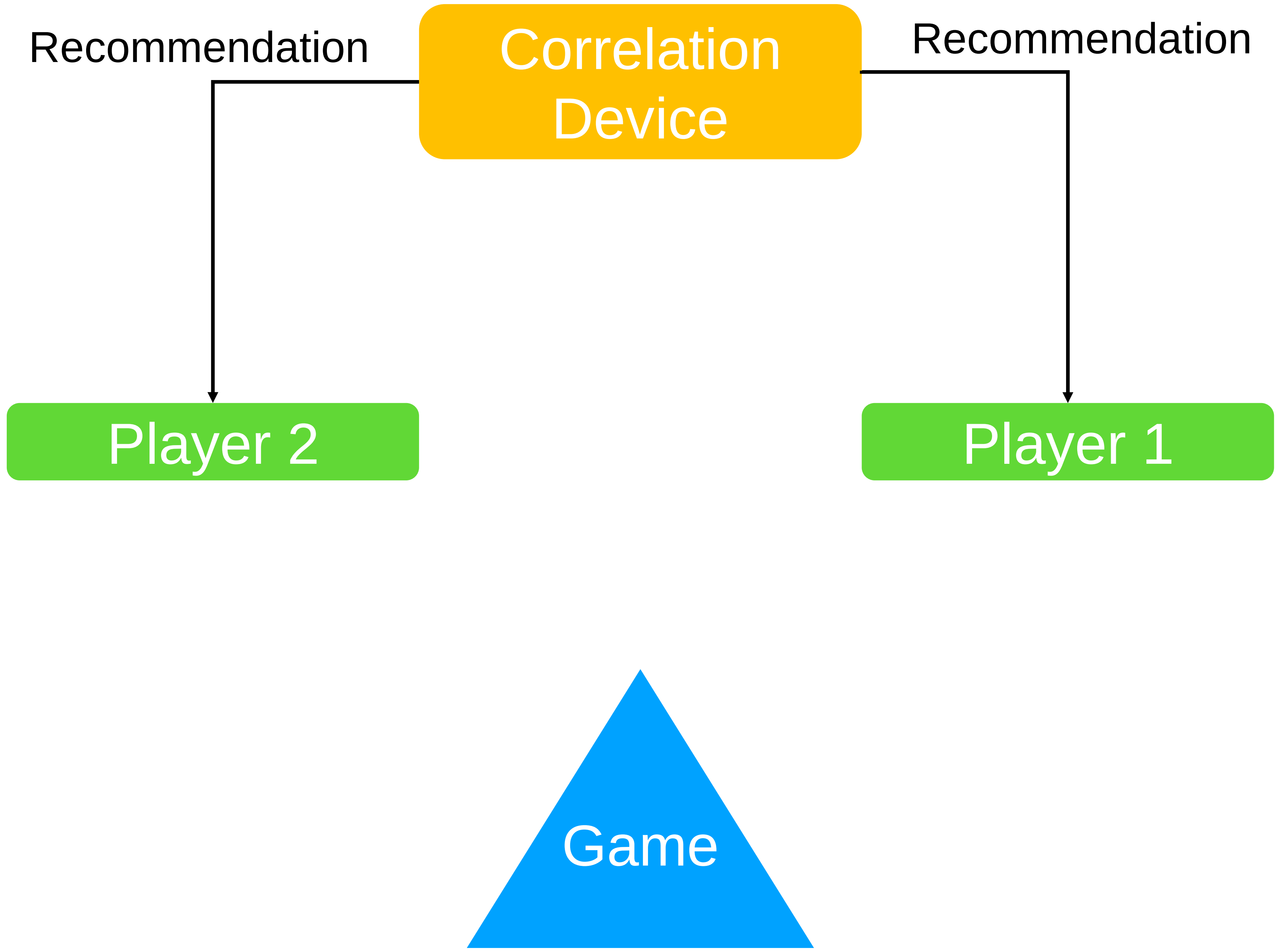
Communication Device:

- *Preplay* and *Intraplay* communication
- *TMEcom*



Correlation Device:

- *Preplay* communication

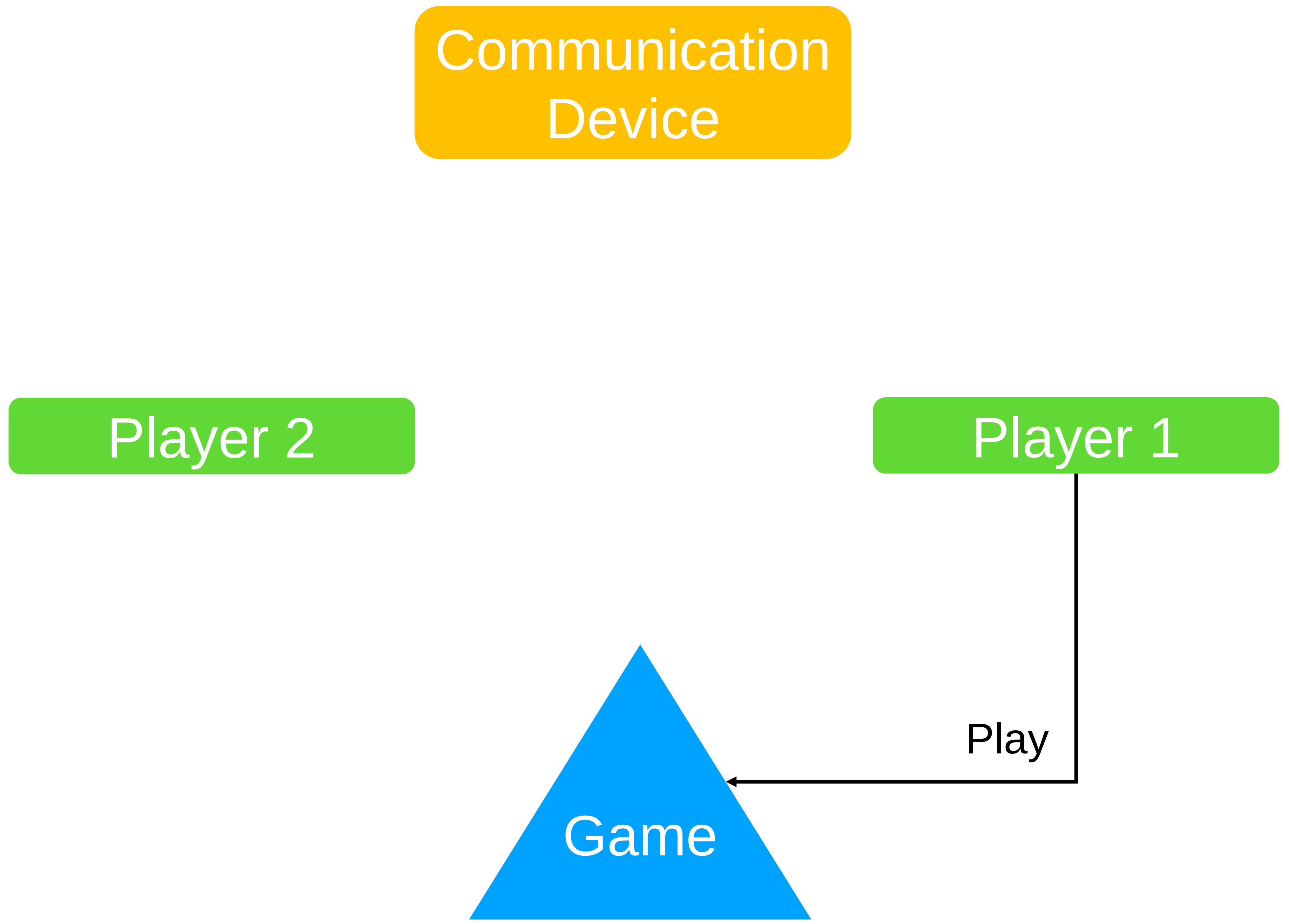


State of the Art: N vs. 1

Two different models of communication between team members, supported by different devices:

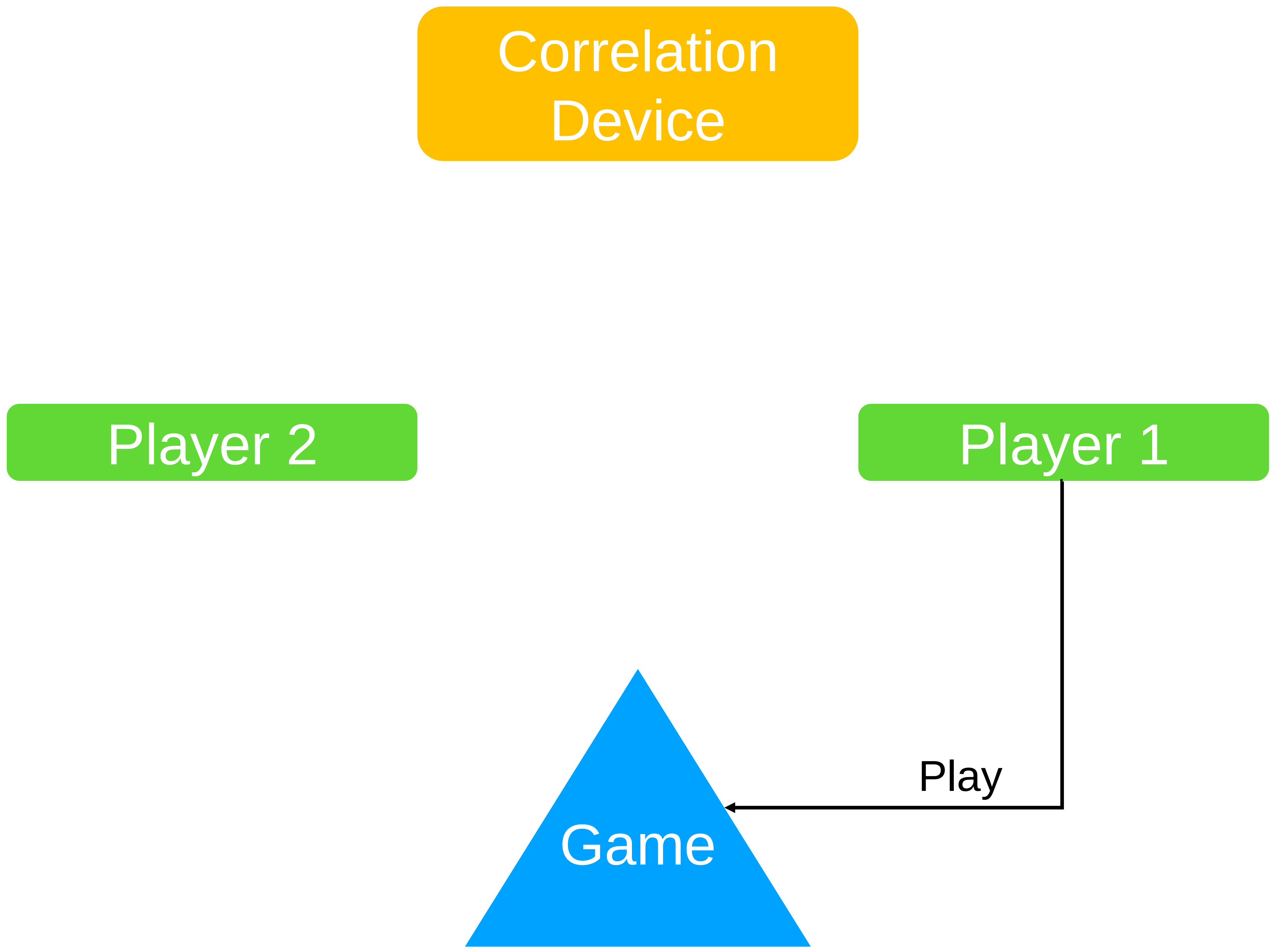
Communication Device:

- *Preplay* and *Intraplay* communication
- *TMEcom*



Correlation Device:

- *Preplay* communication

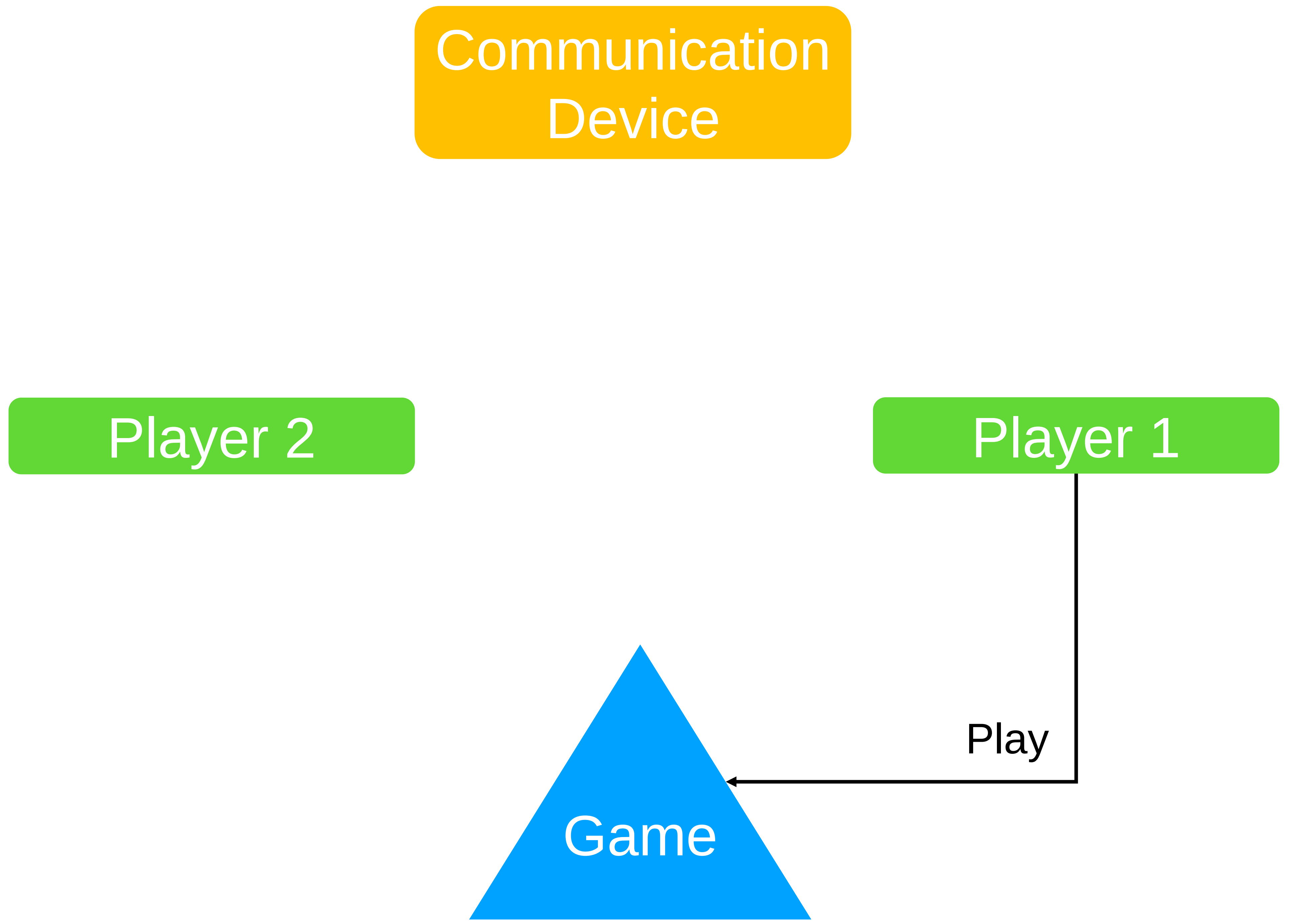


State of the Art: N vs. 1

Two different models of communication between team members, supported by different devices:

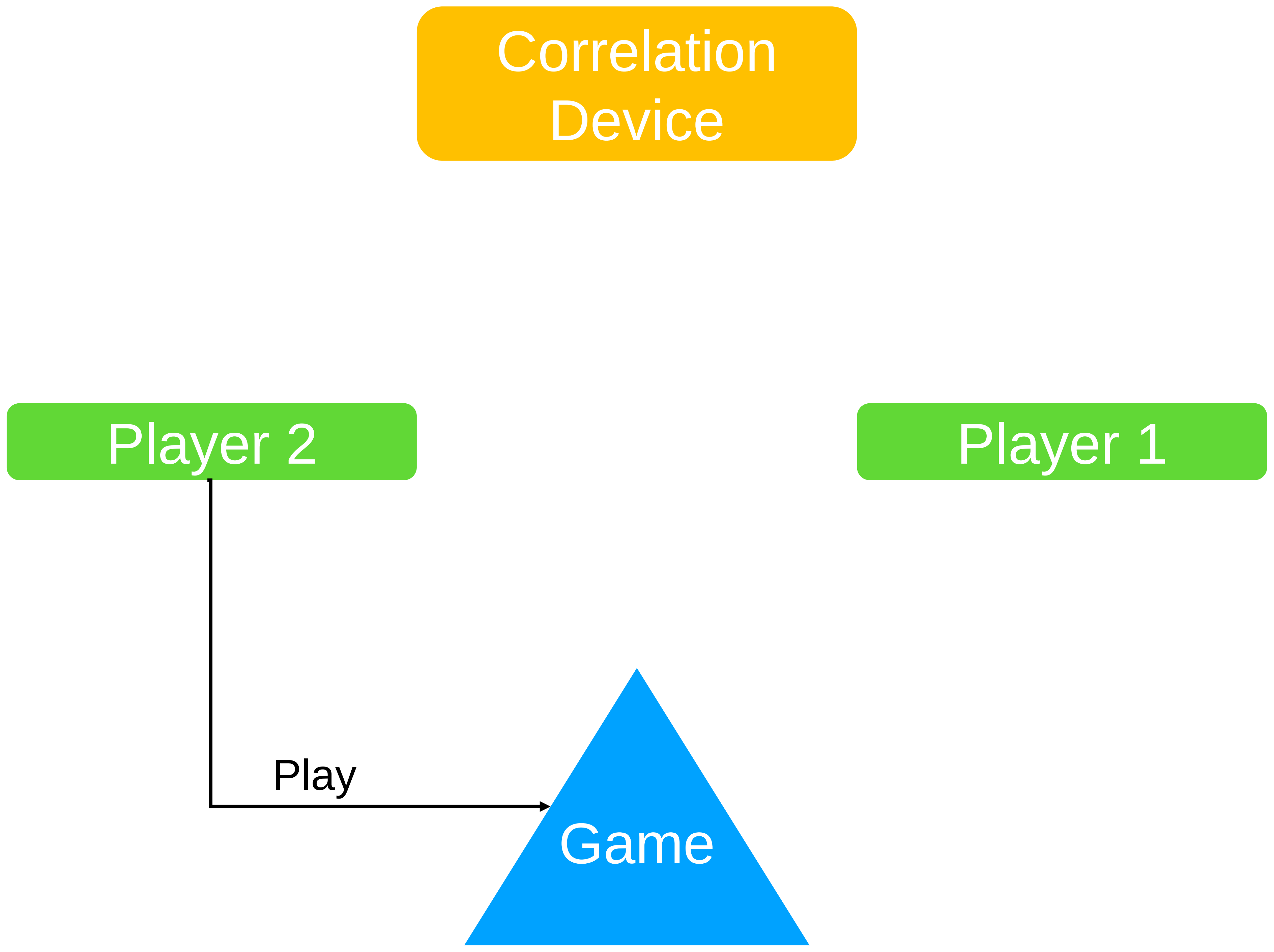
Communication Device:

- *Preplay* and *Intraplay* communication
- *TMEcom*



Correlation Device:

- *Preplay* communication

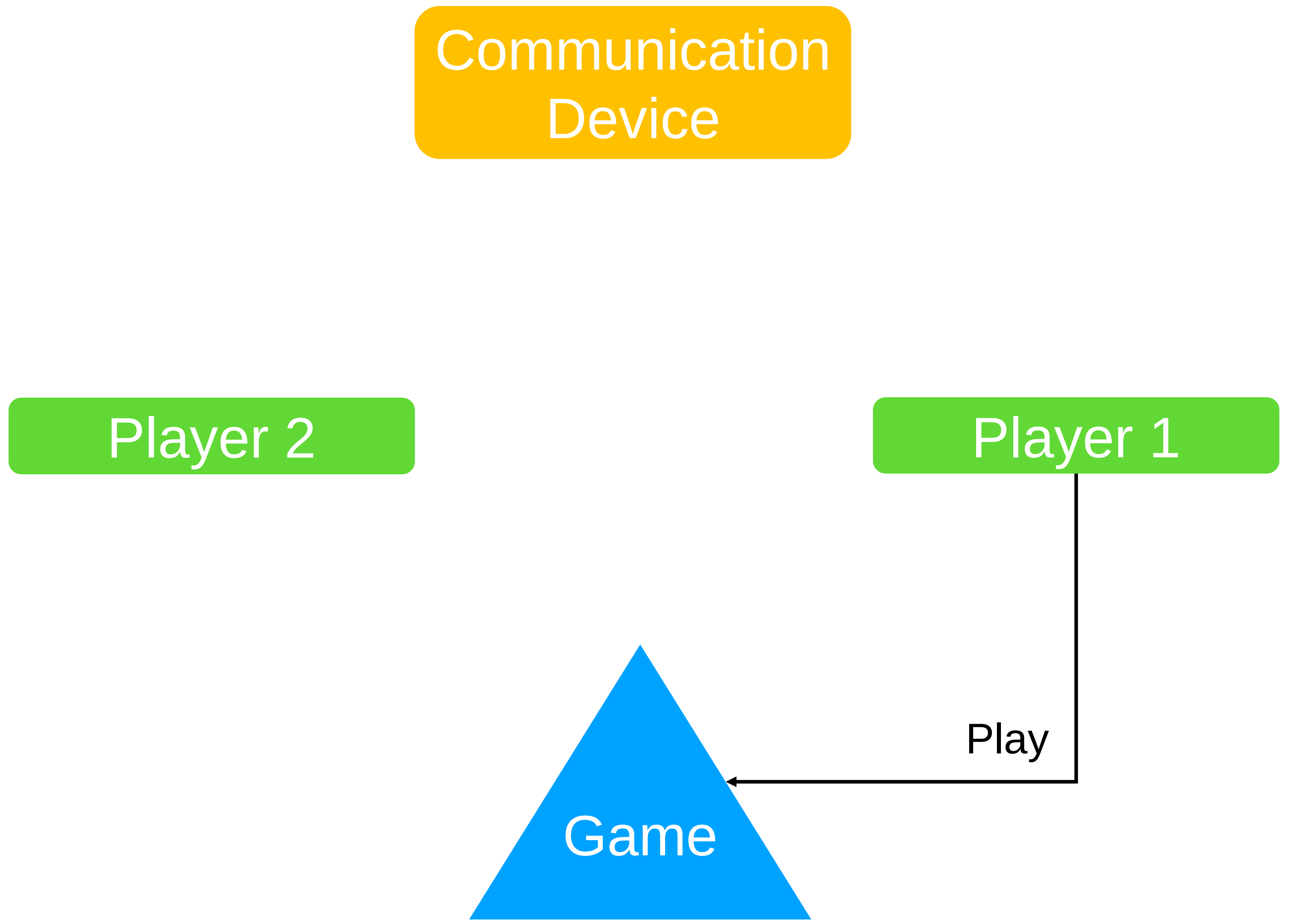


State of the Art: N vs. 1

Two different models of communication between team members, supported by different devices:

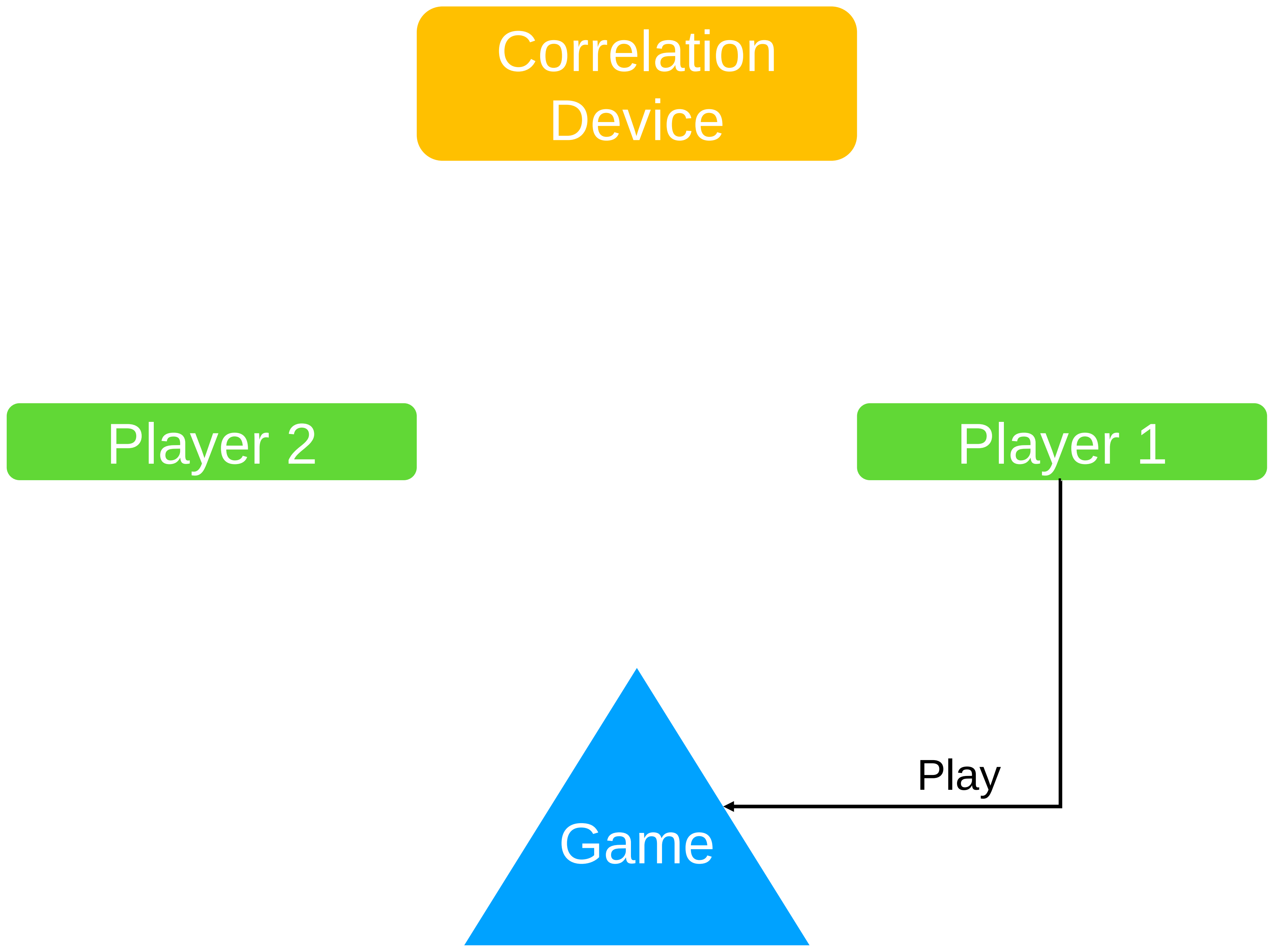
Communication Device:

- *Preplay* and *Intraplay* communication
- *TMEcom*



Correlation Device:

- *Preplay* communication

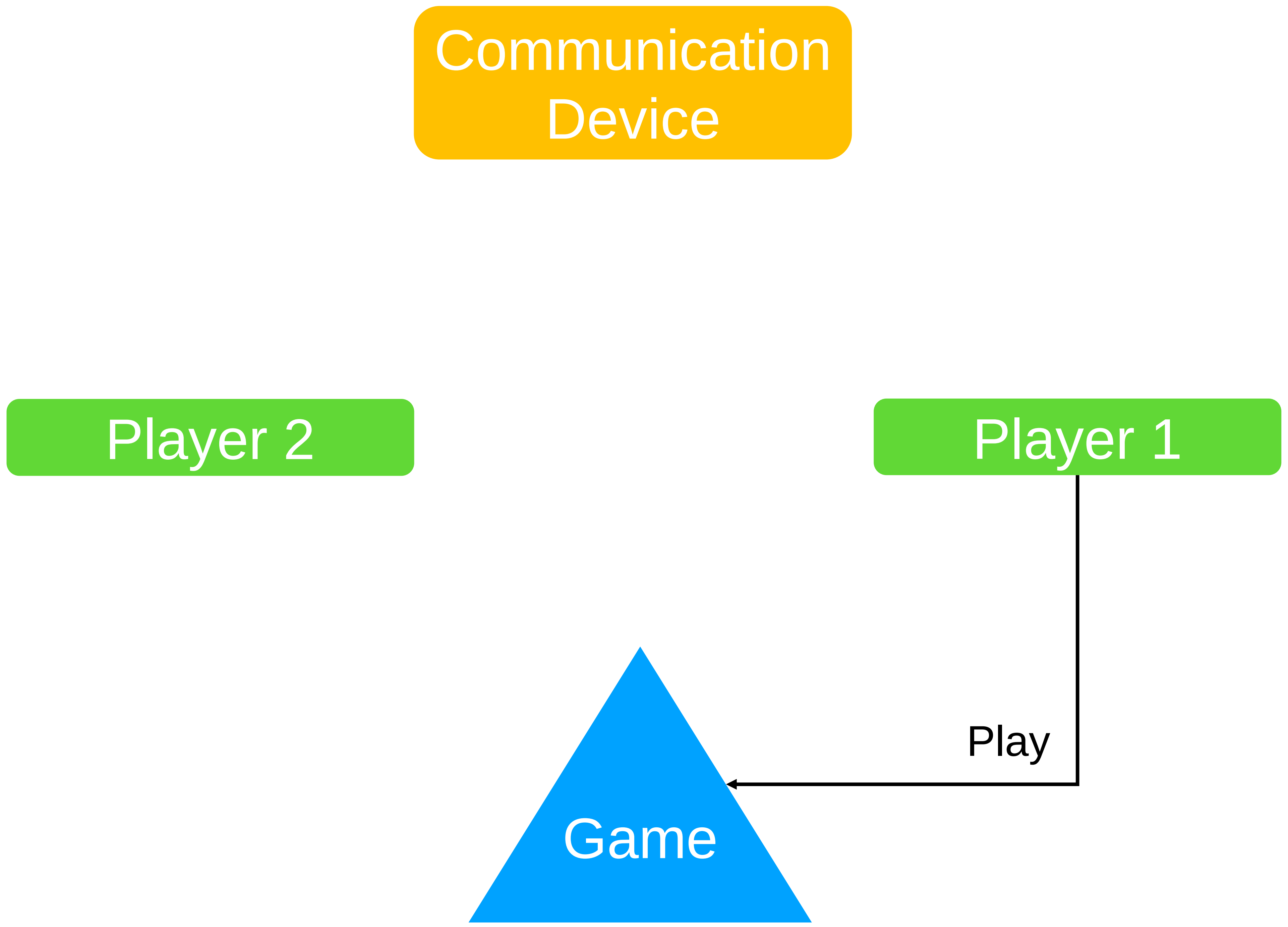


State of the Art: N vs. 1

Two different models of communication between team members, supported by different devices:

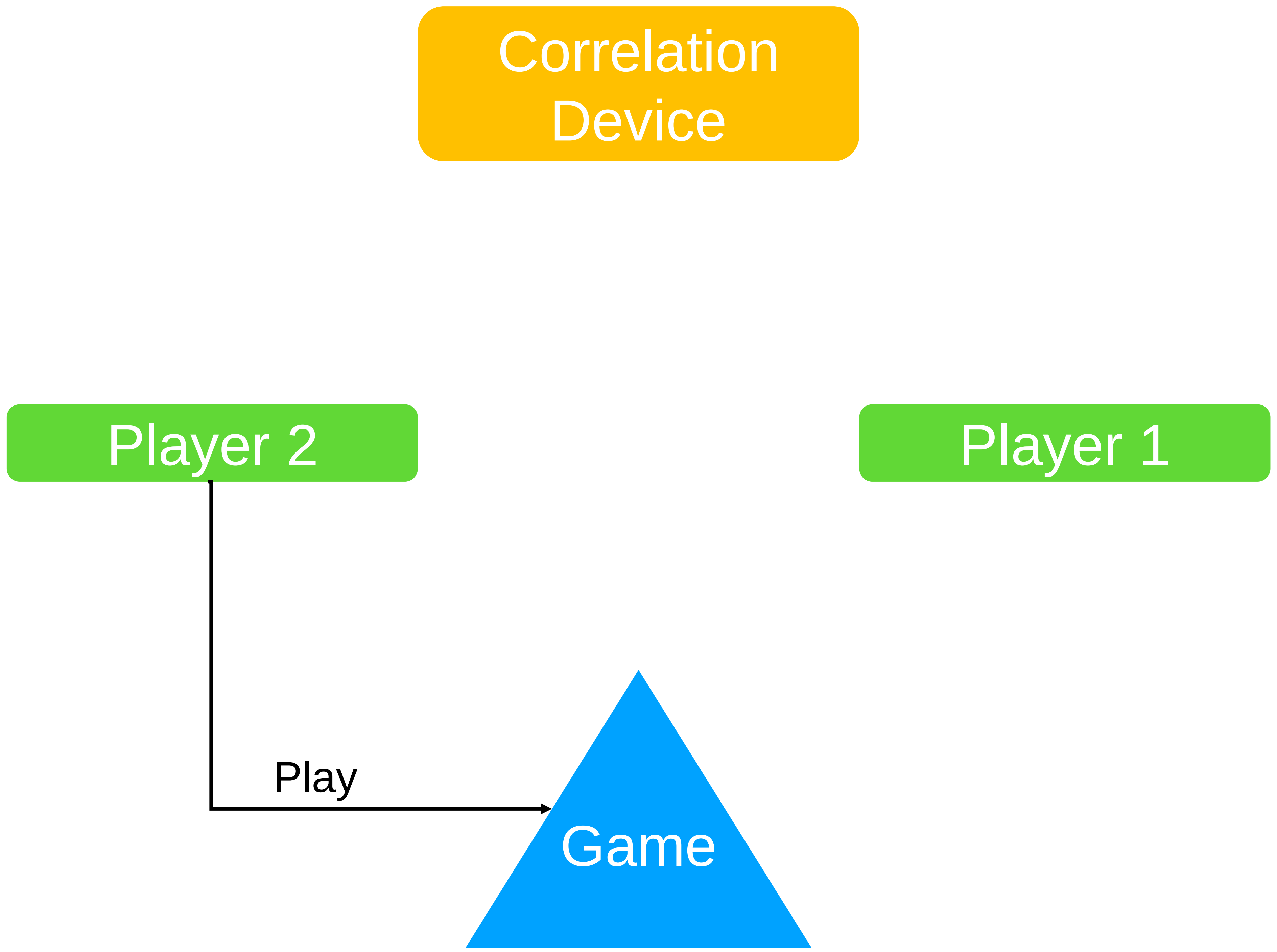
Communication Device:

- *Preplay* and *Intraplay* communication
- *TMEcom*



Correlation Device:

- *Preplay* communication

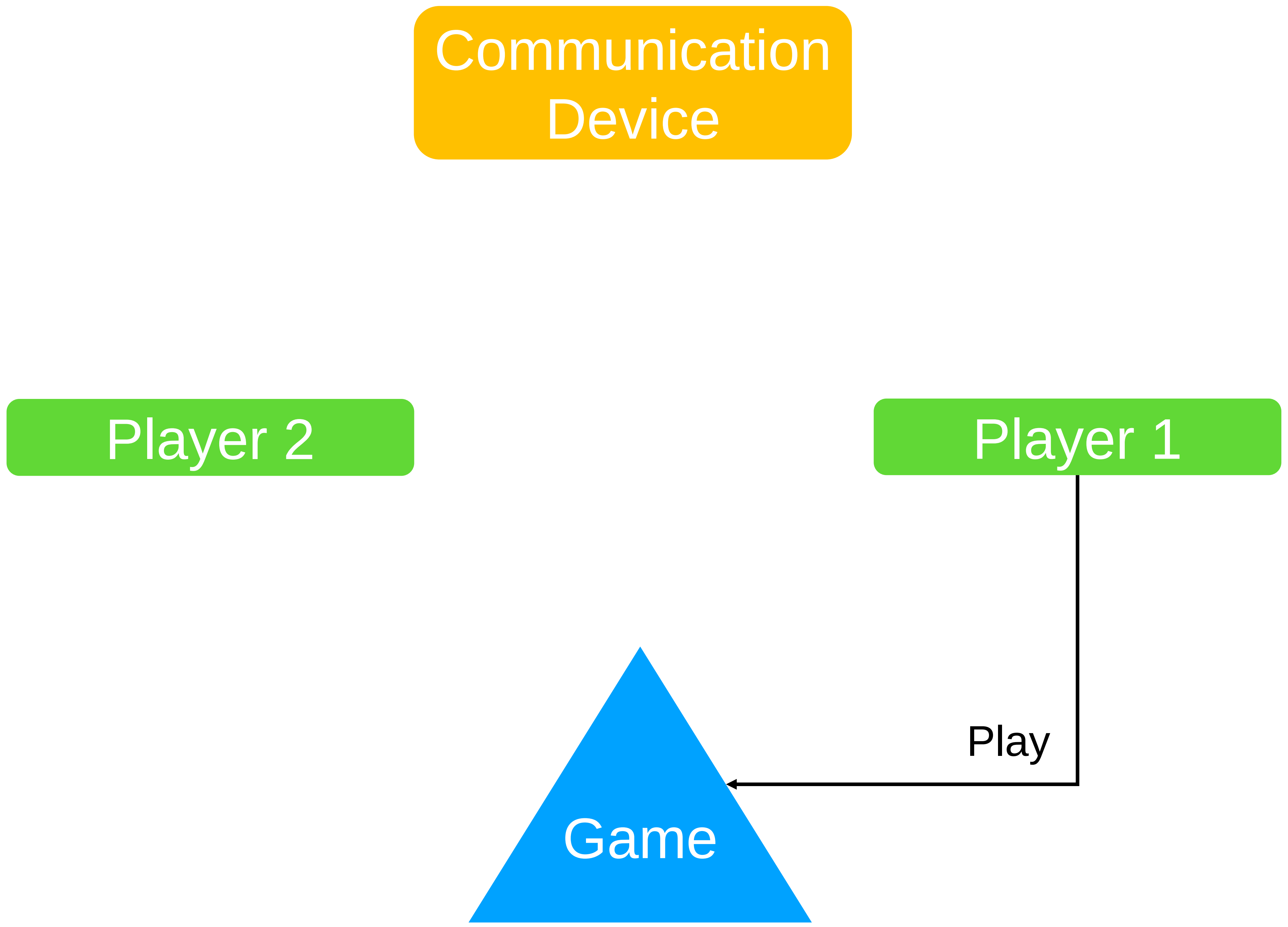


State of the Art: N vs. 1

Two different models of communication between team members, supported by different devices:

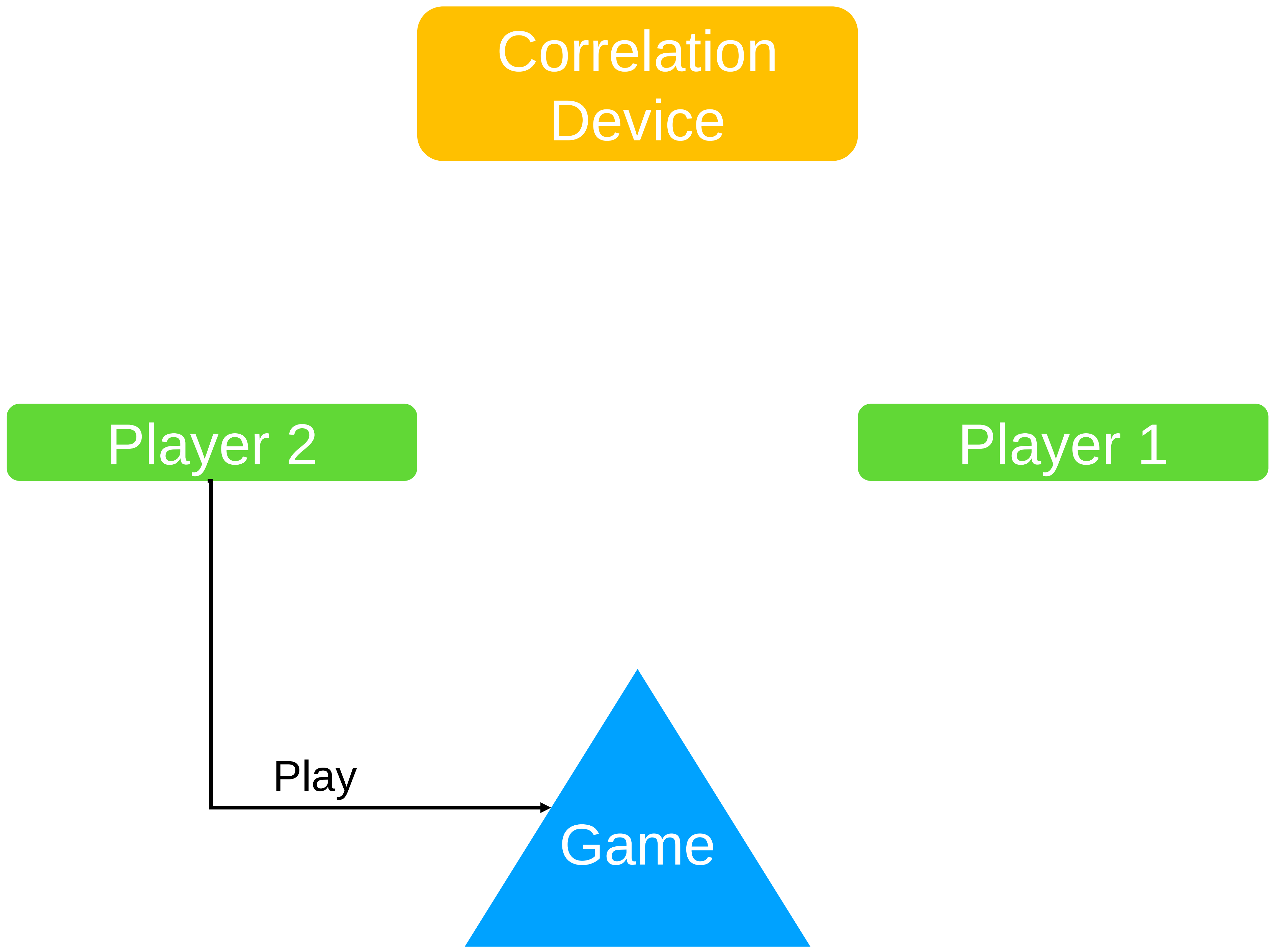
Communication Device:

- *Preplay* and *Intraplay* communication
- *TMEcom*



Correlation Device:

- *Preplay* communication
- *TMEcor*

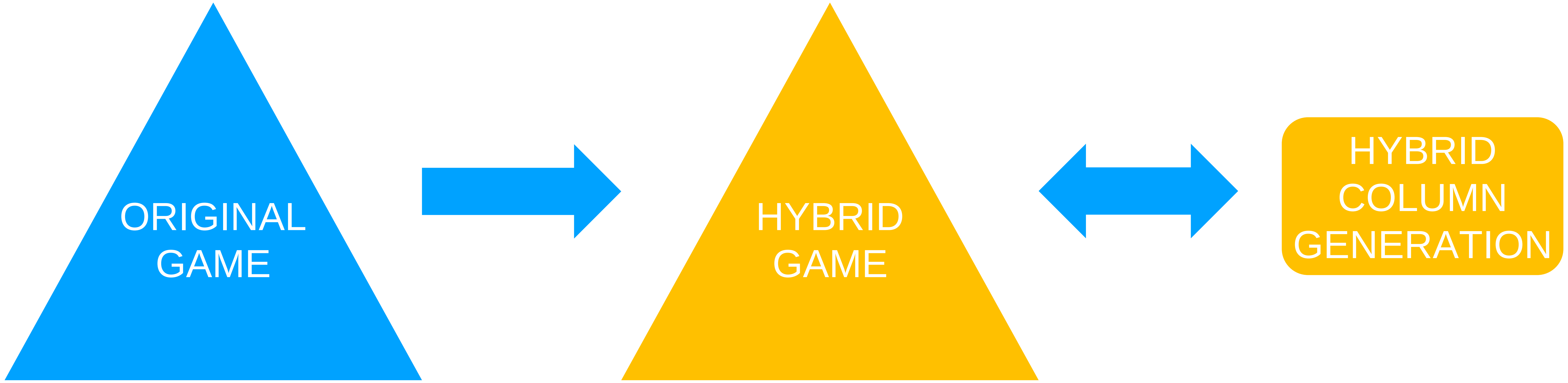


State of the Art: N vs. 1

- TMEcom:
  - Can be computed in polynomial time. (Celli and Gatti, 2017)
  - Requires *intraplay* and *preplay* communication (often not feasible)
- TMEcor:
  - NP-hard. (Celli and Gatti, 2017)
  - Requires only *preplay* communication (almost always feasible)

State of the Art: N vs. 1

Hybrid Column Generation (Celli and Gatti, 2017)

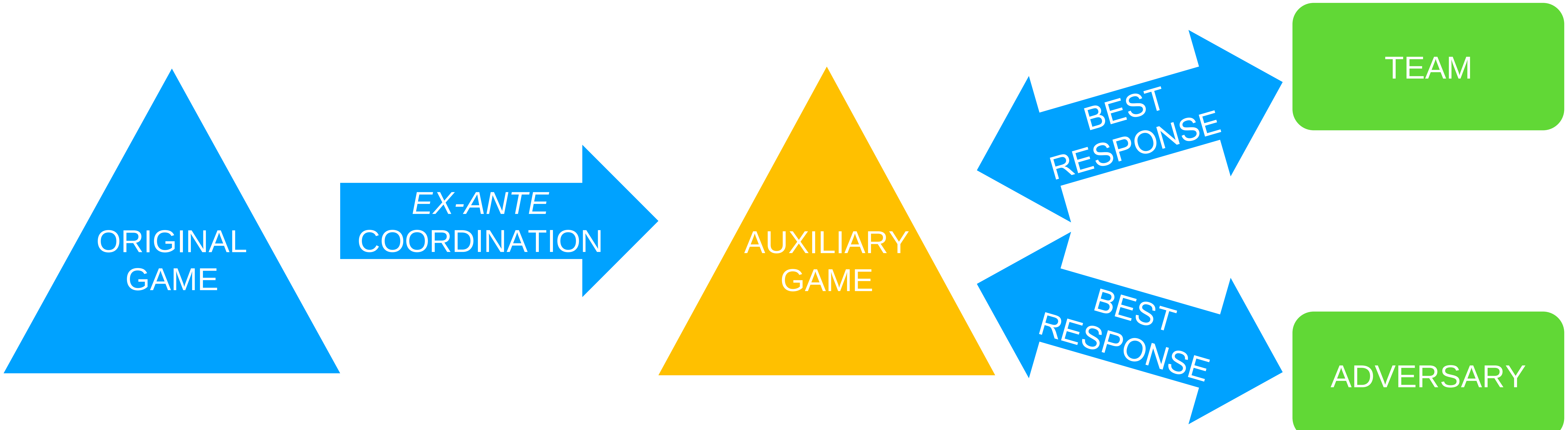


HCG algorithm:

- Two LPs formulated on a progressively larger hybrid formulation of the game
- BR oracle formulated as an ILP
- Approximation can be obtained by relaxing binary constraints of BR oracle
- ILP limits scalability

State of the Art: N vs. 1

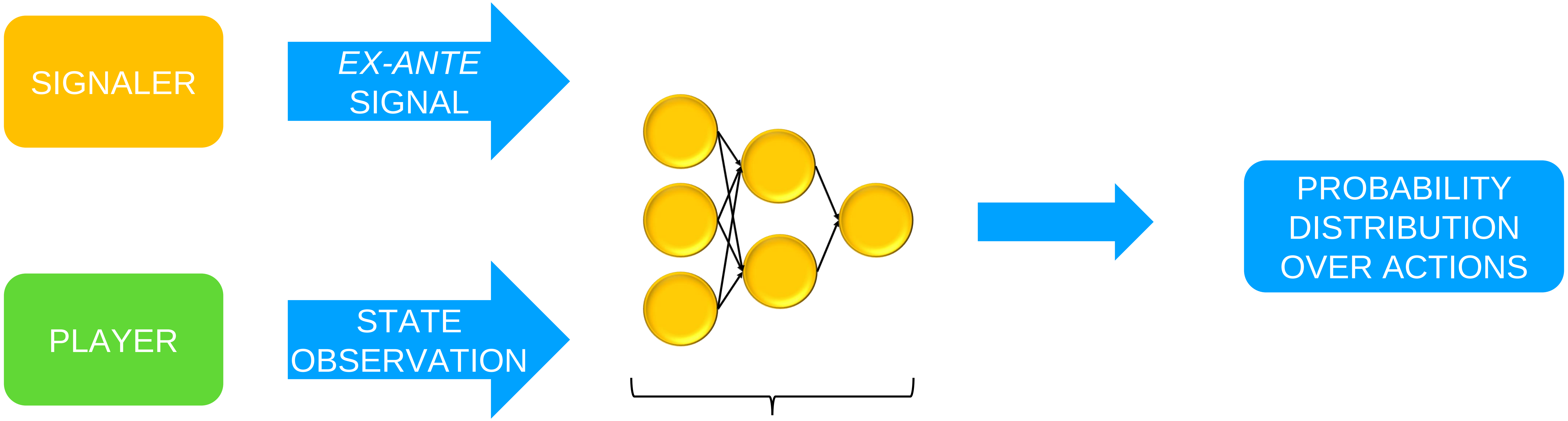
Fictitious Team Play (Farina et al., 2018)



- Best response as a MILP
- Converges to TMEcor (equivalence between NE in auxiliary game and TMEcor in original game)
- MILP limits scalability
- Convergence rate of FP

State of the Art: N vs. 1

Soft Team Actor-Critic (Celli et al., 2019)



Policy Network (actor) + Hypernetworks for parameters generation

- Model-free (no knowledge of the game-tree required)
- Actor-critic architecture with separate policy and value function networks
- Policies are conditioned on an exogenous signal drawn *ex-ante*
- TMEcor approximation under specific assumptions

Outline

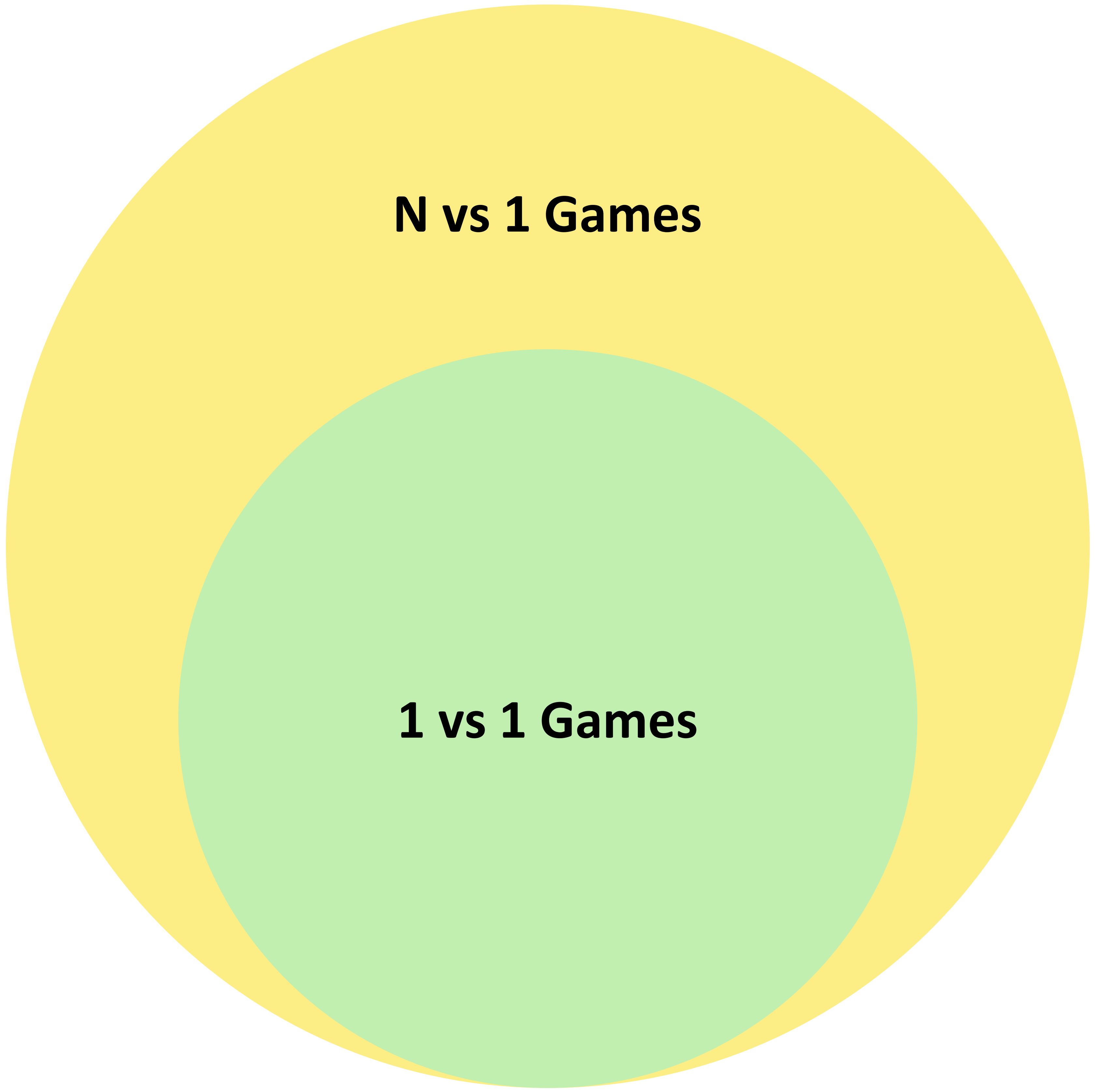
1. Introduction to Algorithmic Game Theory
2. Preliminaries
3. State of the art
4. **Project proposal**

Our Goal

Our goal is to develop scalable and efficient algorithm to find equilibria in the context of team games, offering some theoretical guarantees of convergence

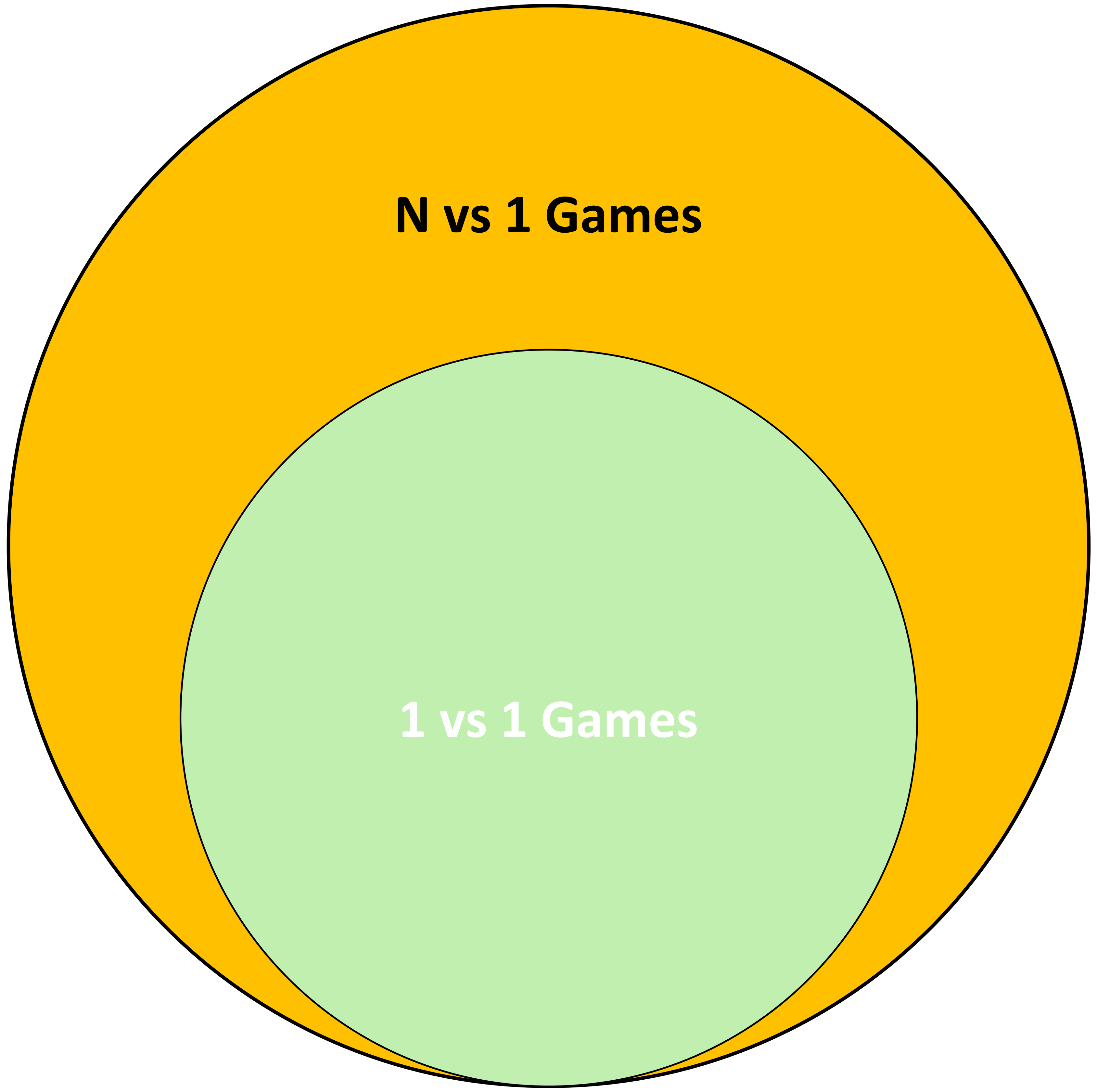
Our Goal

Our goal is to develop scalable and efficient algorithm to find equilibria in the context of team games, offering some theoretical guarantees of convergence



Our Goal

Our goal is to develop scalable and efficient algorithm to find equilibria in the context of team games, offering some theoretical guarantees of convergence



State-of-the-art solutions

- Consider Team as a single player and apply two-players solutions:

State-of-the-art solutions

- Consider Team as a single player and apply two-players solutions:
    - Team strategy grows exponentially with the number of teammates, not scalable

State-of-the-art solutions

- Consider Team as a single player and apply two-players solutions:
  - Team strategy grows exponentially with the number of teammates, not scalable
  - In games with imperfect information team becomes an imperfect recall player

State-of-the-art solutions

- ~~Consider Team as a single player and apply two-players solutions:~~ ✕

State-of-the-art solutions

- ~~Consider Team as a single player and apply two-players solutions:~~ ✕

- Apply model-based solutions:

State-of-the-art solutions

- ~~Consider Team as a single player and apply two-players solutions:~~ ✗

- Apply model-based solutions:
  - Limitations in scalability

State-of-the-art solutions

- ~~Consider Team as a single player and apply two-players solutions:~~ ✕

- ~~Apply model-based solutions:~~ ✕

State-of-the-art solutions

- ~~Consider Team as a single player and apply two-players solutions:~~ ✗

- ~~Apply model-based solutions:~~ ✗

- Apply model-free solutions (e.g. STAC):

State-of-the-art solutions

- ~~Consider Team as a single player and apply two-players solutions:~~ ✗

- ~~Apply model-based solutions:~~ ✗

- Apply model-free solutions (e.g. STAC):
  - No guarantees of convergence

State-of-the-art solutions

- ~~Consider Team as a single player and apply two-players solutions:~~ ✕

- ~~Apply model-based solutions:~~ ✕

- ~~Apply model-free solutions (e.g. STAC):~~ ✕

State-of-the-art solutions

- ~~Consider Team as a single player and apply two-players solutions:~~ ✗

- ~~Apply model-based solutions:~~ ✗

- ~~Apply model-free solutions (e.g. STAC~~ ✗

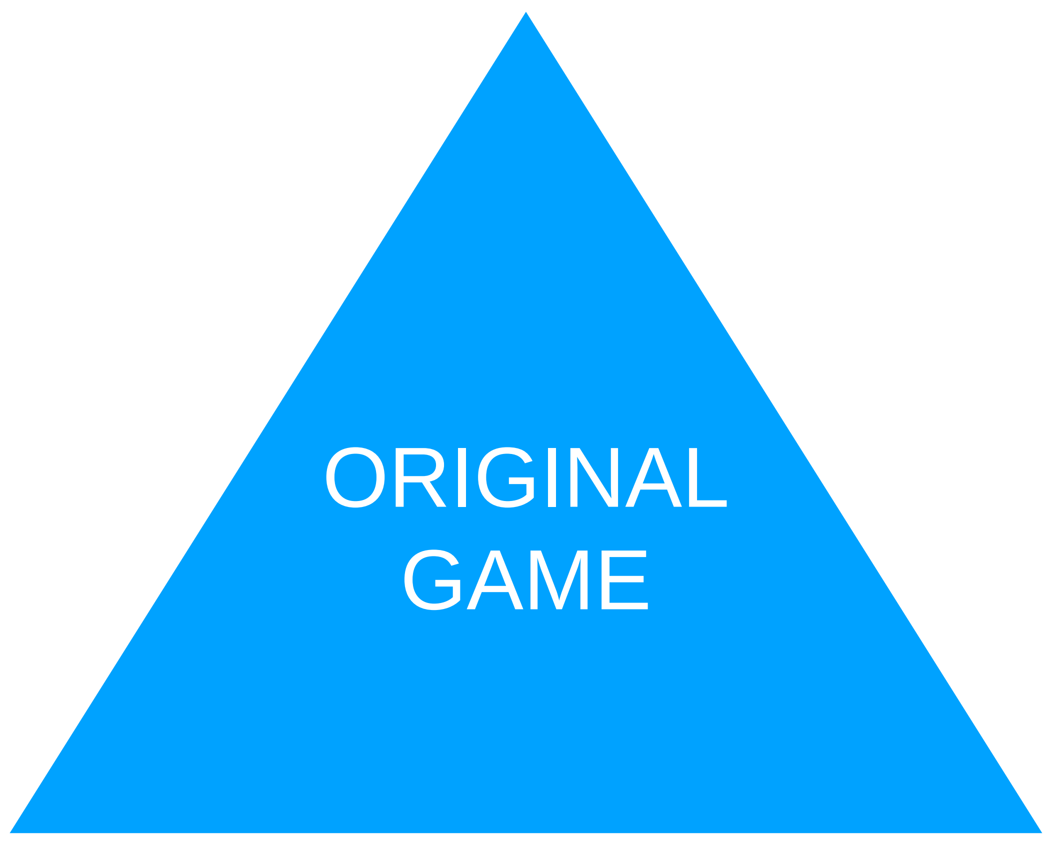
Solution: Use an hybrid approach to gain advantages of different frameworks

Project Proposal

Adapt CFR-BR to the case team vs single opponent:

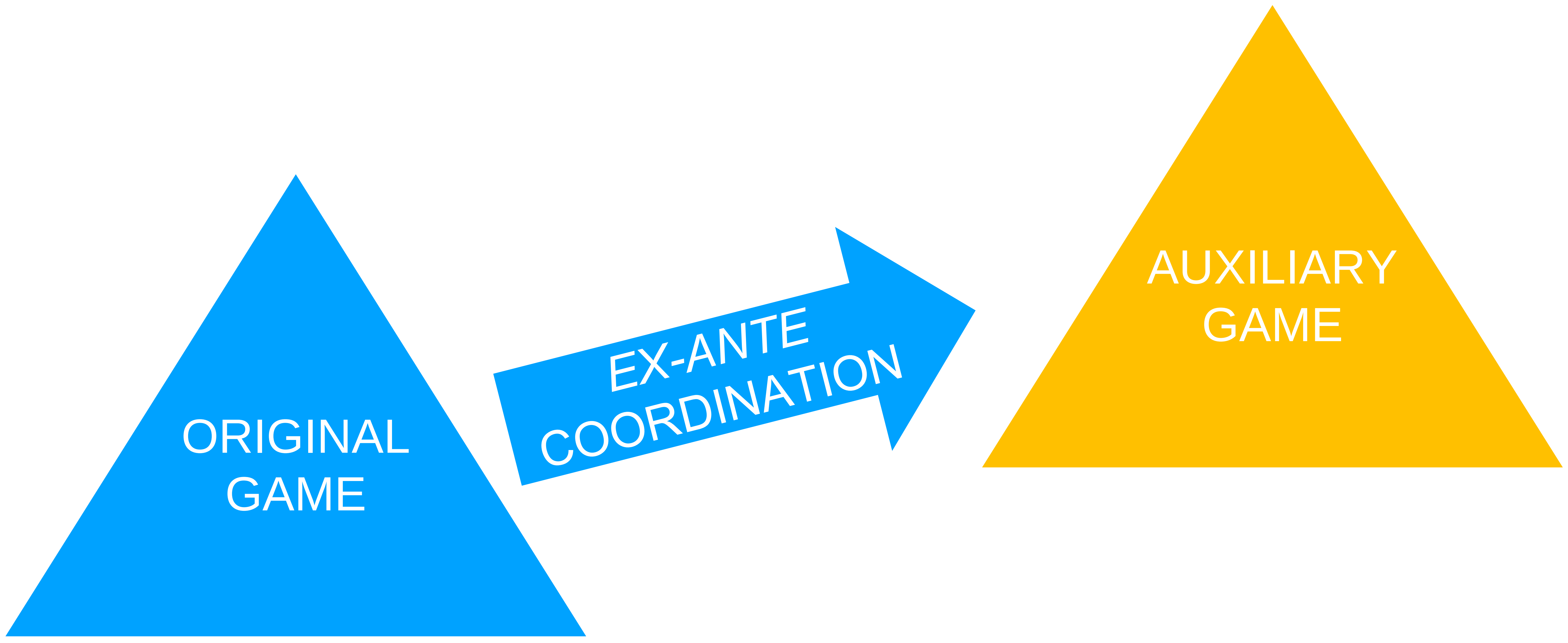
Project Proposal

Adapt CFR-BR to the case team vs single opponent:



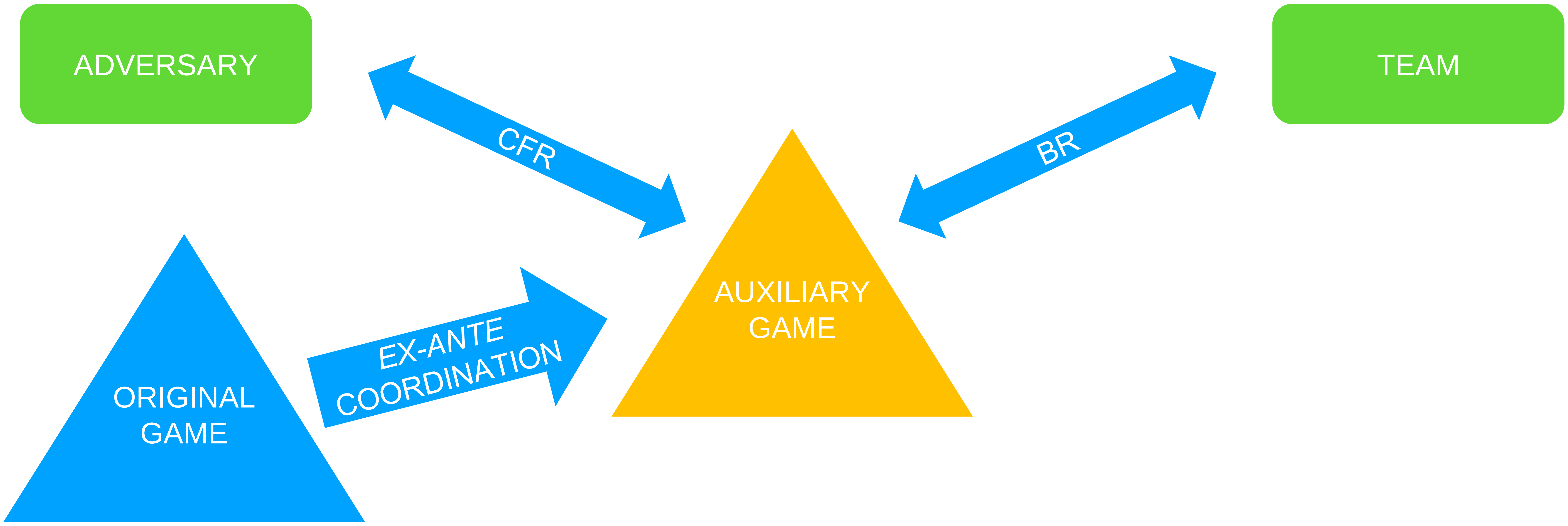
Project Proposal

Adapt CFR-BR to the case team vs single opponent:



Project Proposal

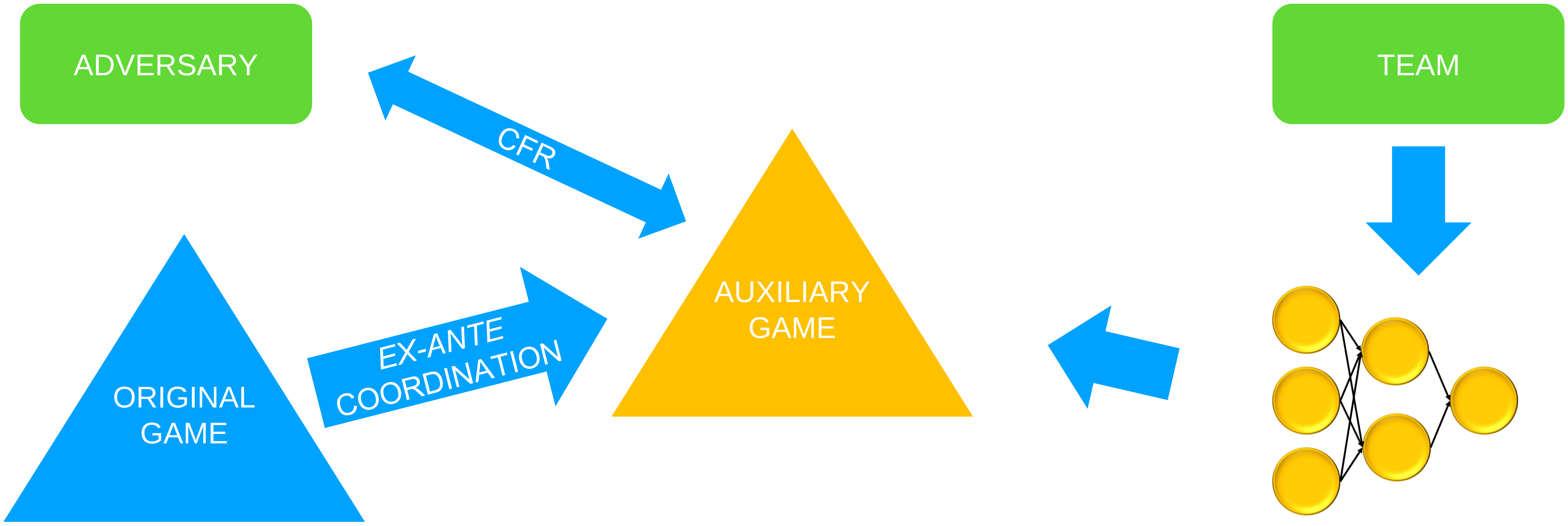
Adapt CFR-BR to the case team vs single opponent:



Project Proposal

Adapt CFR-BR to the case team vs single opponent:

- Approximate BR using Deep Reinforcement Learning



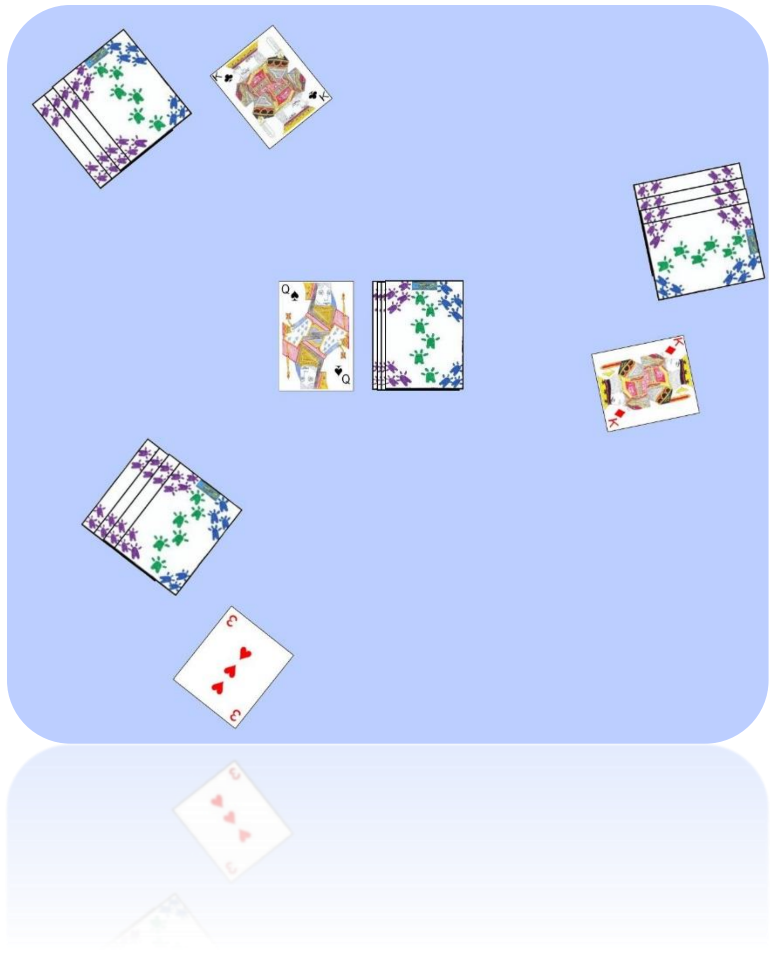
Project Proposal

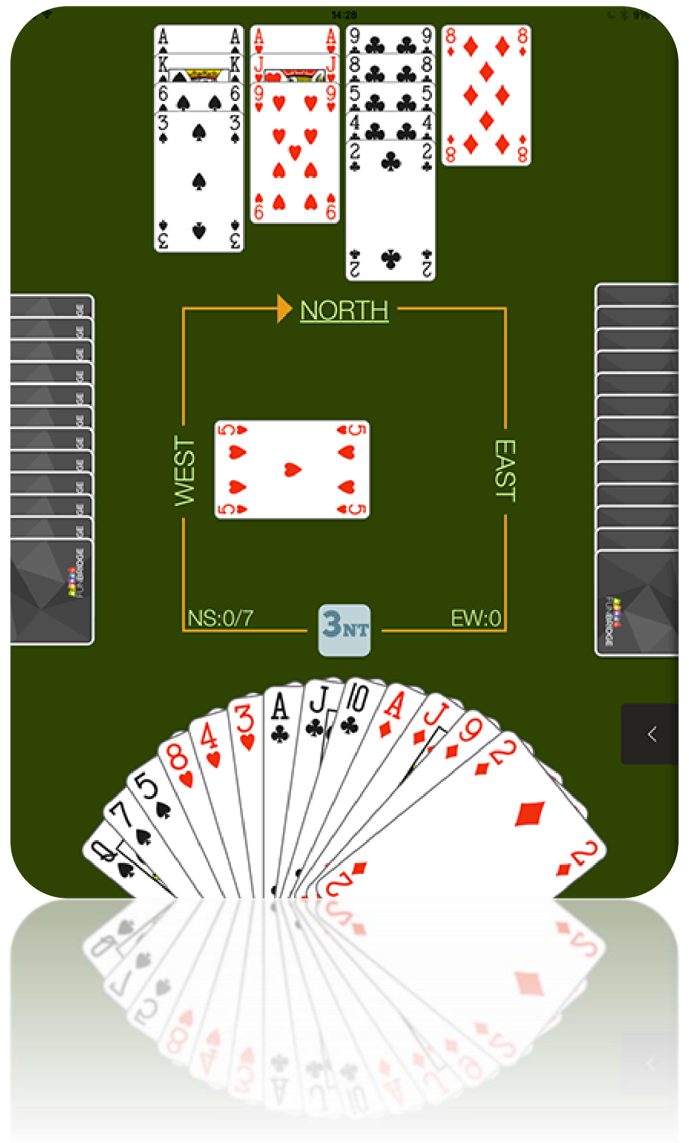
Advantages of the proposed framework:

- Scalability: ✓
  - Represent compactly the team strategy
  - Use a ML approach to solve a problem that is NP-hard (best response)
- Maintain theoretical guarantees proper of CFR ✓

Applications

- Recreational games:
  - Goofspiel
  - Contract bridge
- Real-world:
  - Security
  - Car races





Questions?

Thank You For Your Attention!