Research Project Proposal: Efficient Solutions for Adversarial Team Games

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CSE Track

POLITECNICO MILANO 1863

HP-SR in Information Technology
Outline

1. Introduction to Algorithmic Game Theory
2. Preliminaries
3. State of the art
4. Project proposal
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1. Introduction to Algorithmic Game Theory
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3. State of the art
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“Game theory is the name given to the methodology of using mathematical tools to model and analyze situations of interactive decision making. These are situations involving several decision makers (called players) with different goals, in which the decision of each affects the outcome for all the decision makers.”


- Algorithmic Game Theory is the area at the intersection between Game Theory and Computer Science
Recreational Games
Recreational Games
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Potential Real-World Applications

- **Physical Security**: Strategic organization of the available resources

- **Car Races**: Coordination of strategies among team members
Outline

1. Introduction to Algorithmic Game Theory

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4. Project proposal
Game Tree
Game Tree

Player 1

Player 2
Game Tree

Player 1

Player 2
Game Tree
Game Tree

Player 1

Player 2

\[ R \quad L \]

\[ r \quad l \quad r \quad l \]
Game Tree

\[\begin{align*}
R & \quad L \\
(6,6) & \quad (0,7) & \quad (7,0) & \quad (1,1)
\end{align*}\]
Game Tree

Player 1

Player 2

<table>
<thead>
<tr>
<th></th>
<th>r</th>
<th>l</th>
</tr>
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<tbody>
<tr>
<td>R</td>
<td>(6,6)</td>
<td>(0,7)</td>
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Game Tree

Player 1

Player 2

\[(6,6)\]
\[(0,7)\]
\[(7,0)\]
\[(1,1)\]

\[r\]
\[l\]

\[R\]
\[L\]

\[0.75\]
\[0.25\]

\[r\]
\[l\]

\[(6,6)\]
\[(0,7)\]

\[(7,0)\]
\[(1,1)\]
Game Tree

Player 1

Player 2

<table>
<thead>
<tr>
<th></th>
<th>0.5</th>
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<tbody>
<tr>
<td>r</td>
<td>0.75</td>
<td>r</td>
</tr>
<tr>
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Branches:
- Player 1 chooses between right (R) and left (L).
- Player 2 observes the choice and chooses between right (r) and left (l).
- The payoff matrix is shown with probabilities and outcomes.
Information and recall in games

Perfect vs. imperfect information game

In some games, defined as perfect information games, the state of the game is completely observable by the players.

When the state is not completely observable, the game is defined as imperfect information game.

Perfect vs. imperfect recall game

A perfect recall game is a game in which no player forgets information that he/she acquired before.

An imperfect recall game is a game in which there is at least a player that is an imperfect recall player (e.g. it forgets some information that was known before in the game).
Team

- A team is a set of players that share the same objectives in the game.

- In Game Theory a team is modeled as a set of players that have the same utility function.
Nash Equilibrium

- Solution concept introduced by John Nash in 1951
Nash Equilibrium

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- A NE is a joint combination of strategies stable with respect to unilateral deviations of a single player
Nash Equilibrium

• Solution concept introduced by John Nash in 1951

• A NE is a joint combination of strategies stable with respect to unilateral deviations of a single player

\[
\begin{array}{c|cc}
& r & l \\
\hline
R & (6,6) & (0,7) \\
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\end{array}
\]
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• Approximation of Nash Equilibrium ($\varepsilon$-NE): joint combination of strategies such that no player can gain more than $\varepsilon$ by unilaterally deviating
Team Maxmin Equilibrium

- Team Maxmin Equilibrium is the NE that maximizes the team utility

- From the team’s perspective, a generic NE can be arbitrarily inefficient w.r.t. the TME
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State of the art

1 vs 1 Games
(polynomial complexity)
State of the art

N vs 1 Games (NP-hard)

1 vs 1 Games (polynomial complexity)
State of the art

N vs 1 Games (NP-hard)

1 vs 1 Games (polynomial complexity)

Introduce state-of-the-art algorithms for two-player games

Explain source of NP-hardness of solving team games

Introduce state-of-the-art algorithms for team games
State of the art: 1 vs. 1 games

Fictitious Play: (Slow convergence rate)

\[ \epsilon \sim O\left(\frac{1}{\sqrt{T}}\right) \]

- Fictitious Play (FP) (Brown, 1951)
- Weakened FP (van der Genugten, 2000)
- Fictitious Self Play (Heinrich et al., 2015)
- Neural FSP (Brown et al., 2018)

No-regret learning: (Fast convergence rate)

\[ \epsilon \sim O\left(\frac{1}{\sqrt{T}}\right) \]

- Monte Carlo CFR (Lanctot et al., 2009)
- CFR-BR (Johanson et al., 2012)
- Counterfactual Regret Minimization (CFR) (Zinkevich et al., 2008)
- Deep-CFR (Brown et al., 2018)
State of the Art: N vs. 1

• From the perspective of a team, not correlating the strategies of the teammates can be inefficient, in a measure depending on the number of players and on the number of available actions (Basilico et al., 2016)

• Focus on 2 vs 1 games
State of the Art: N vs. 1
Two different models of communication between team members, supported by different devices:
State of the Art: N vs. 1

Two different models of communication between team members, supported by different devices:

Communication Device:
State of the Art: N vs. 1

Two different models of communication between team members, supported by different devices:

Communication Device:
• Preplay and Intraplay communication
Two different models of communication between team members, supported by different devices:

Communication Device:
- *Preplay* and *Intraplay* communication
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Two different models of communication between team members, supported by different devices:

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- TMEcom
State of the Art: N vs. 1

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Communication Device:
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Correlation Device:
State of the Art: N vs. 1

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- *TMEcom*

Correlation Device:
- *Preplay* communication

Diagram:
- **Communication Device**
  - Player 2
  - Player 1
  - Game

- **Correlation Device**
  - Recommendation
  - Player 2
  - Player 1
  - Game
State of the Art: N vs. 1

Two different models of communication between team members, supported by different devices:

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State of the Art: N vs. 1

Two different models of communication between team members, supported by different devices:

Communication Device:
- *Preplay* and *Intraplay* communication
- *TMEcom*

Correlation Device:
- *Preplay* communication

---

**Communication Device**

- Player 1
- Player 2
- Game

**Correlation Device**

- Player 1
- Player 2
- Game
State of the Art: N vs. 1

Two different models of communication between team members, supported by different devices:

Communication Device:
- *Preplay* and *Intraplay* communication
- *TMEcom*

Correlation Device:
- *Preplay* communication

![Diagram showing the communication models for Player 1 and Player 2 during Preplay and Intraplay phases.]
State of the Art: N vs. 1

Two different models of communication between team members, supported by different devices:

**Communication Device:**
- *Preplay* and *Intraplay* communication
- *TMEcom*

**Correlation Device:**
- *Preplay* communication
- *TMEcor*
State of the Art: N vs. 1

• **TMEmcom:**
  - Can be computed in polynomial time. (Celli and Gatti, 2017)
  - Requires *intraplay* and *preplay* communication (often not feasible)

• **TMEmcor:**
  - NP-hard. (Celli and Gatti, 2017)
  - Requires only *preplay* communication (almost always feasible)
State of the Art: N vs. 1

Hybrid Column Generation (Celli and Gatti, 2017)

HCG algorithm:
• Two LPs formulated on a progressively larger hybrid formulation of the game
• BR oracle formulated as an ILP
• Approximation can be obtained by relaxing binary constraints of BR oracle
• ILP limits scalability
State of the Art: N vs. 1

Fictitious Team Play (Farina et al., 2018)

- Best response as a MILP
- Converges to TMEcor (equivalence between NE in auxiliary game and TMEcor in original game)
- MILP limits scalability
- Convergence rate of FP
State of the Art: N vs. 1

Soft Team Actor-Critic (Celli et al., 2019)

- Model-free (no knowledge of the game-tree required)
- Actor-critic architecture with separate policy and value function networks
- Policies are conditioned on an exogenous signal drawn *ex-ante*
- TMEcor approximation under specific assumptions
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Our Goal

Our goal is to develop scalable and efficient algorithm to find equilibria in the context of team games, offering some theoretical guarantees of convergence.
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State-of-the-art solutions

• Consider Team as a single player and apply two-players solutions:
State-of-the-art solutions

- Consider Team as a single player and apply two-players solutions:
  - Team strategy grows exponentially with the number of teammates, not scalable
State-of-the-art solutions

- Consider Team as a single player and apply two-players solutions:
  - Team strategy grows exponentially with the number of teammates, not scalable
  - In games with imperfect information team becomes an imperfect recall player
State-of-the-art solutions

• Consider Team as a single player and apply two-players solutions:
State-of-the-art solutions

- Consider Team as a single player and apply two-players solutions: ❌

- Apply model-based solutions:
State-of-the-art solutions

- Consider Team as a single player and apply two-players solutions:

- Apply model-based solutions:
  - Limitations in scalability
State-of-the-art solutions

- Consider Team as a single player and apply two-players solutions:

- Apply model-based solutions:
State-of-the-art solutions

• Consider Team as a single player and apply two-players solutions: ✗

• Apply model-based solutions: ✗

• Apply model-free solutions (e.g. STAC):
State-of-the-art solutions

• Consider Team as a single player and apply two-players solutions: ✗

• Apply model-based solutions: ✗

• Apply model-free solutions (e.g. STAC):
  • No guarantees of convergence
State-of-the-art solutions

- Consider Team as a single player and apply two-players solutions:

- Apply model-based solutions:

- Apply model-free solutions (e.g., STAC):
State-of-the-art solutions

- Consider Team as a single player and apply two-players solutions: ✗

- Apply model-based solutions: ✗

- Apply model-free solutions (e.g. STAC) ✗

**Solution**: Use an hybrid approach to gain advantages of different frameworks
Project Proposal

Adapt CFR-BR to the case team vs single opponent:
Project Proposal

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Project Proposal

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Adapt CFR-BR to the case team vs single opponent:
Project Proposal

Adapt CFR-BR to the case team vs single opponent:

- Approximate BR using Deep Reinforcement Learning
Advantages of the proposed framework:

- Scalability: ✓
  - Represent compactly the team strategy
  - Use a ML approach to solve a problem that is NP-hard (best response)
  - Maintain theoretical guarantees proper of CFR ✓
Applications

• Recreational games:
  • Goofspiel
  • Contract bridge

• Real-world:
  • Security
  • Car races
Questions?
Thank You For Your Attention!