PUBLIC INFORMATION REPRESENTATION
for Adversarial Team Games

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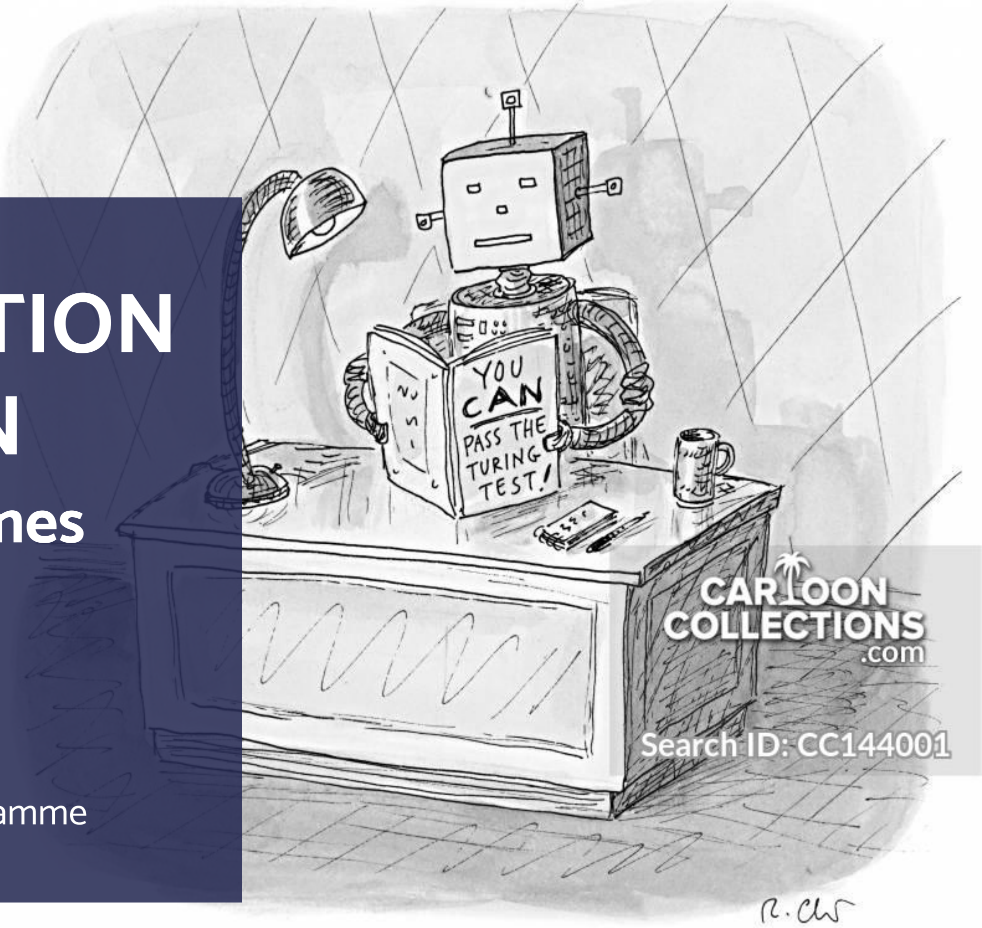

HONOURS PROGRAMME HP-SR in Information Technology

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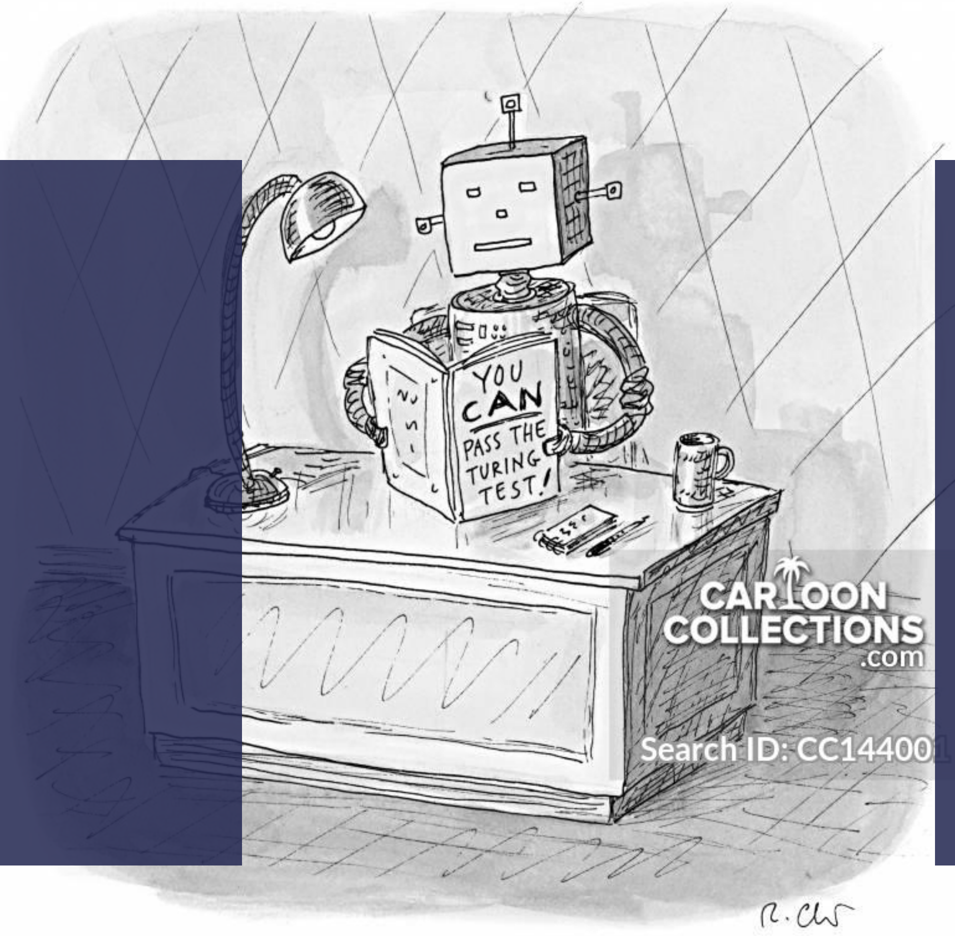
Introduction to Games



FOCUS OF OUR WORK

Multiagent games with mixed cooperative–competitive structure

Multiple agents organized in teams act sequentially on the environment with the goal of maximizing a payoff.
We focus on the **N vs 1** games, also called **adversarial team games**.

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Applications

GAME THEORY

1.Introduction

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We adopt the **mathematical approach** defined by ***Algorithmic Game Theory.***

This allows to formally represent:

- **Games,** that are *sequential interactions of agents*
- **Strategies** of the players, that are *functions associating an action to each decision point*
- **Solution concepts,** that are *rational equilibria for player strategies*

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More efficient and semantic representation for sequential interactions

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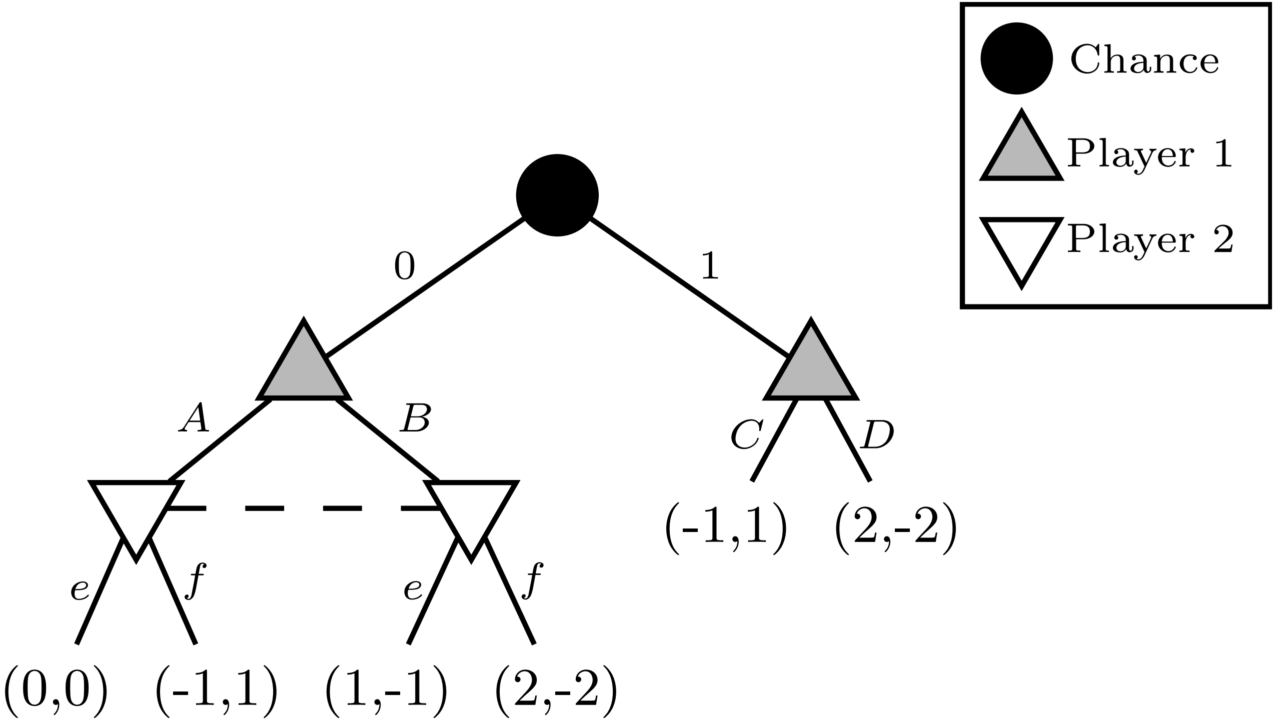
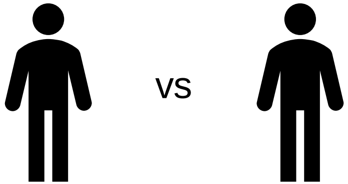
EXAMPLE OF A GAME



TWO-PLAYERS ZERO-SUM GAMES

Two-players Zero-sum games (2p0s) is a simple class of games, in which **any gain for one player corresponds to a loss of the other one**, expressed as opposite payoffs for each terminal node.



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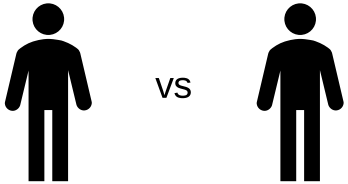

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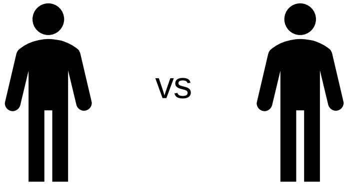

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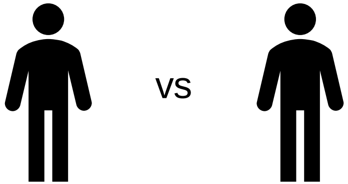

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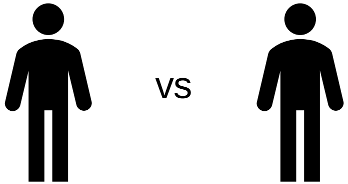

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Properties:

- Finding a Nash Equilibrium is a **P-complete** problem

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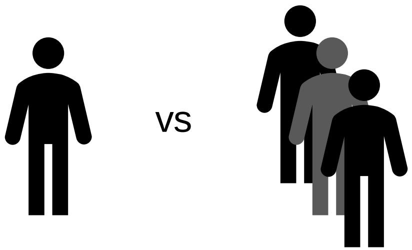

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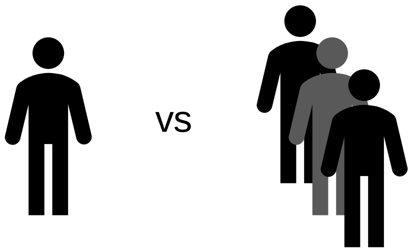

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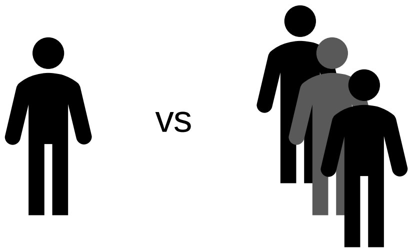

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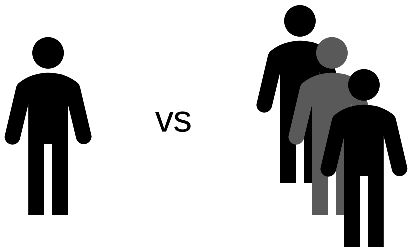

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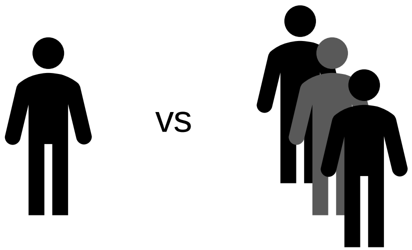

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Properties:

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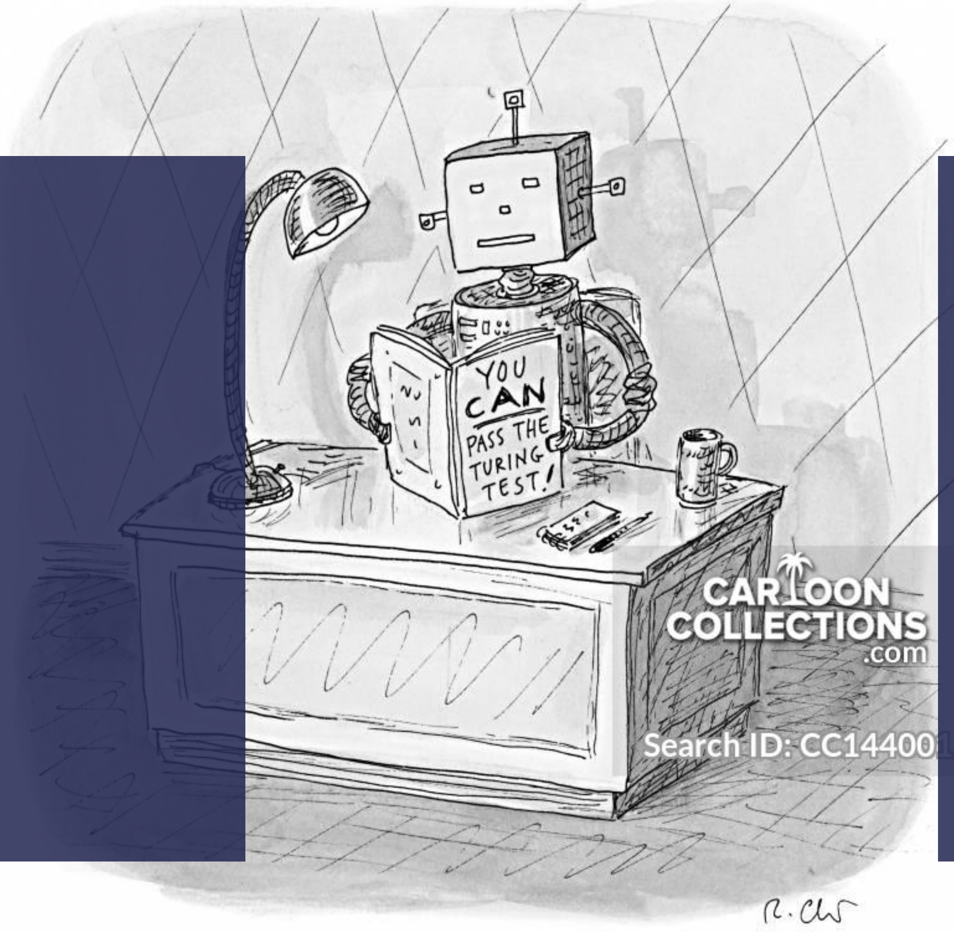
Motivations of our work



SOLVING 2P0S GAMES

Approaches to the solution of 2p0s games

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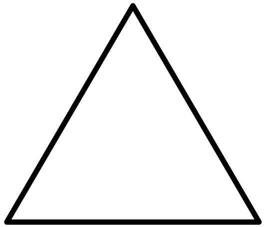

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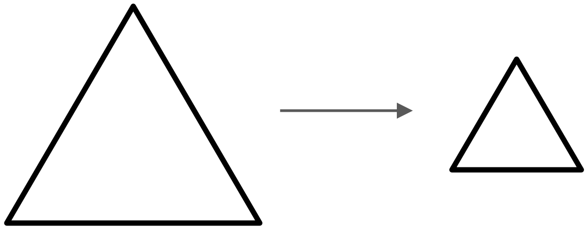

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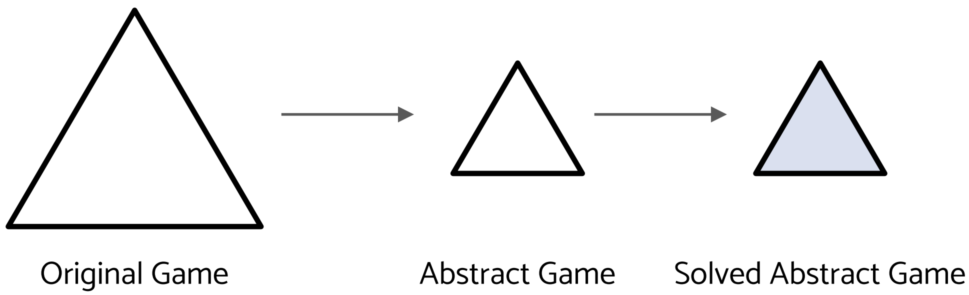

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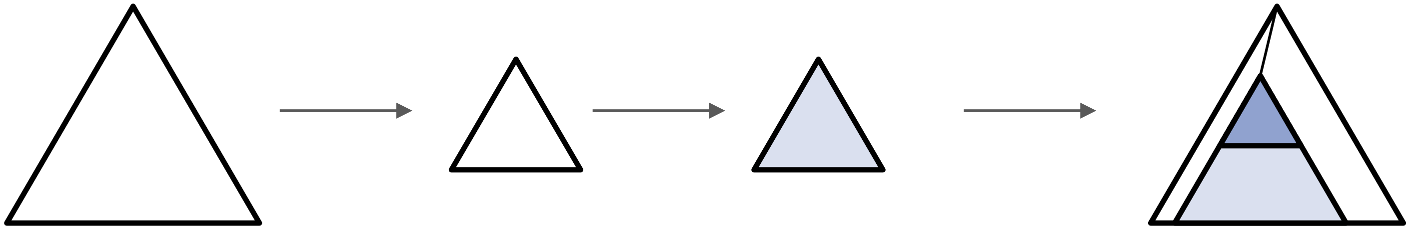

Original Game | Abstract Game | Solved Abstract Game | Continual Resolving at runtime

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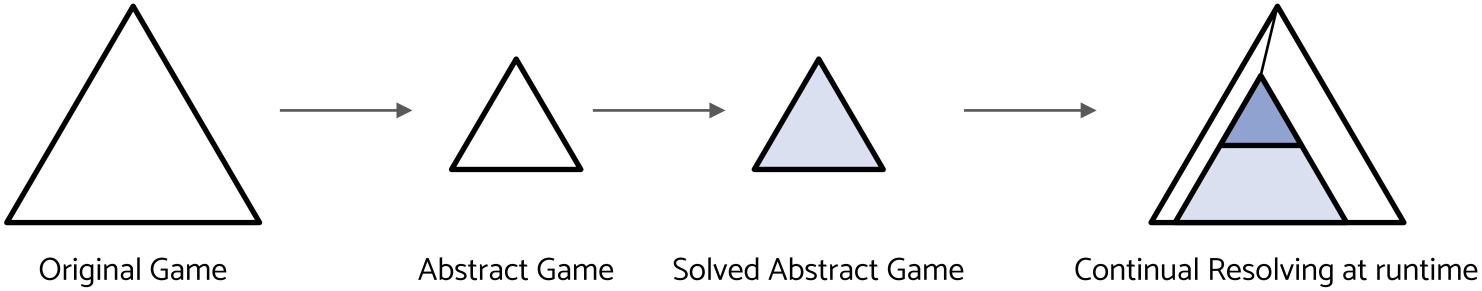
Winning Approach: **Superhuman perfomances in Poker**



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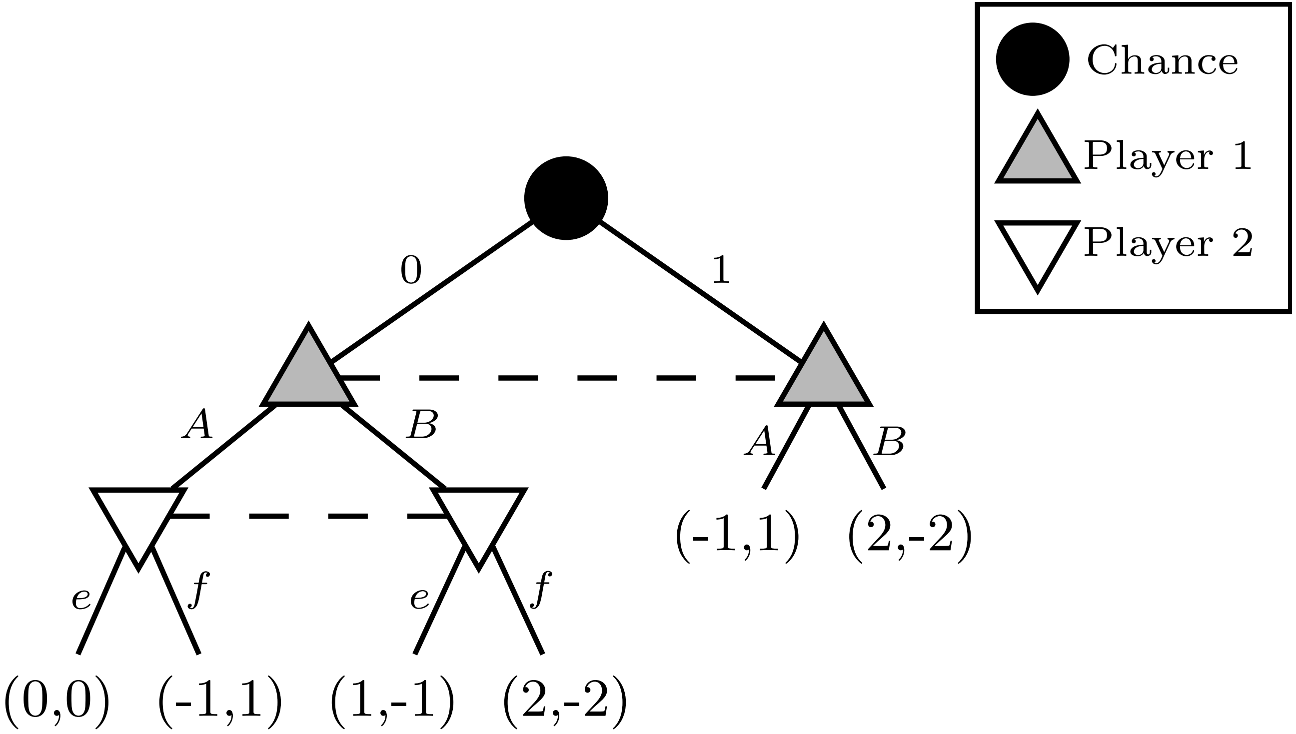

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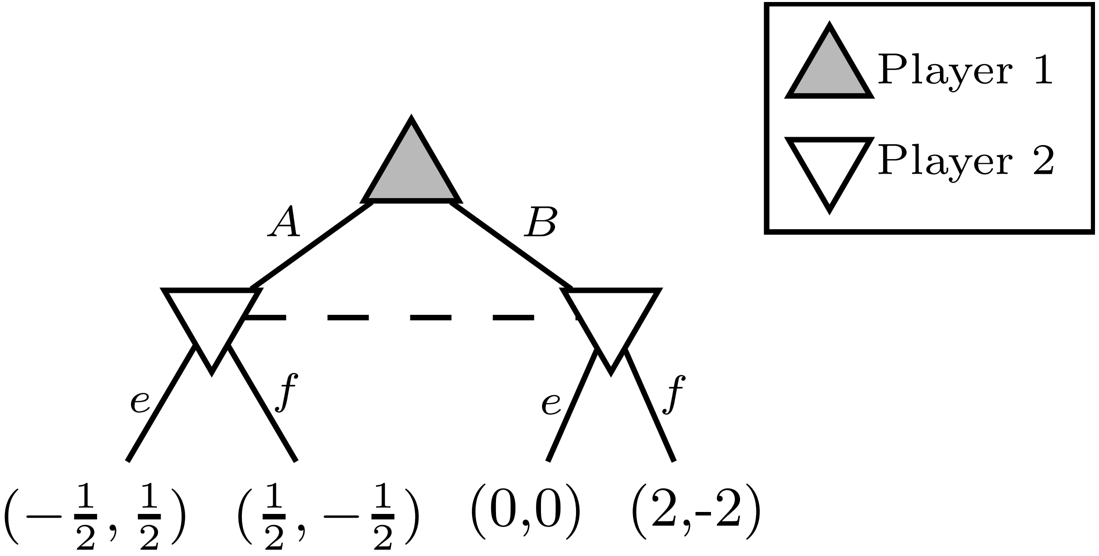

SOLVING ADVERSARIAL TEAM GAMES

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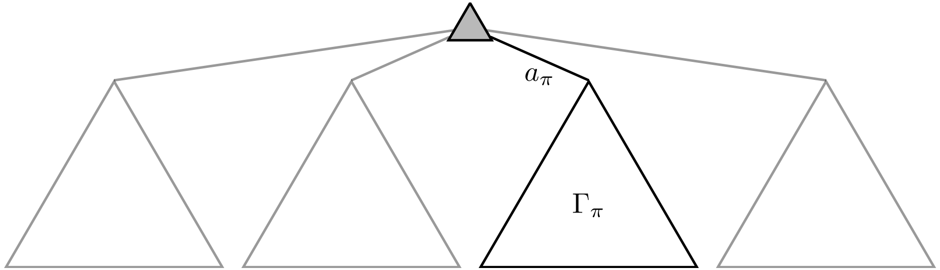

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π is a plan for both players
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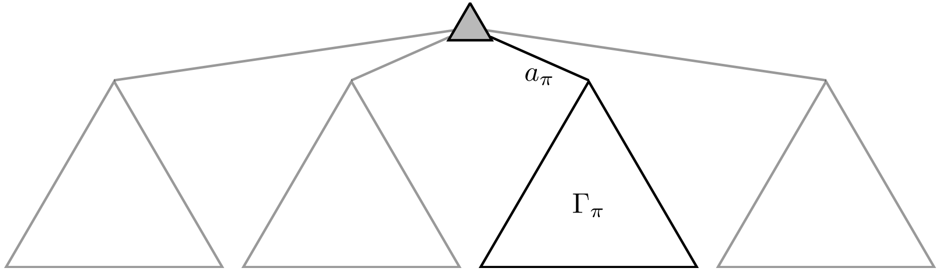

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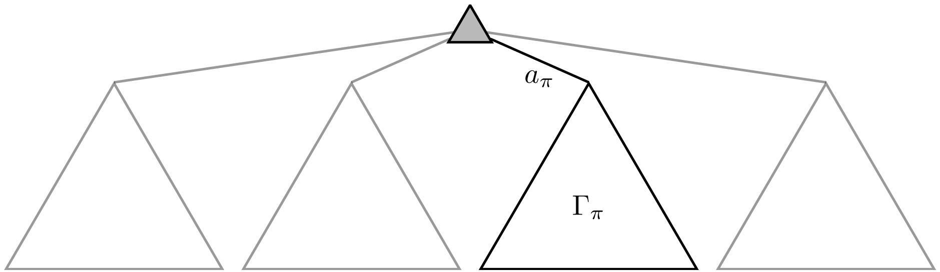

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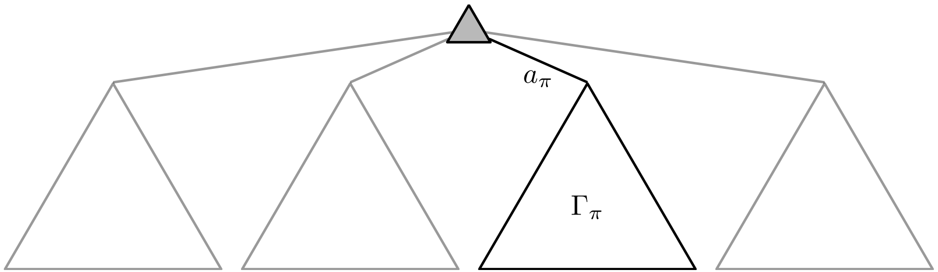

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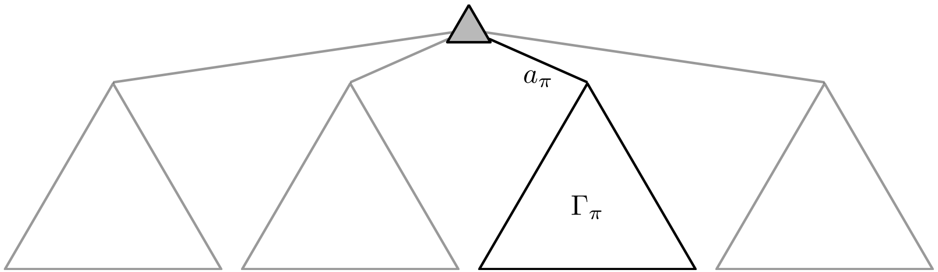

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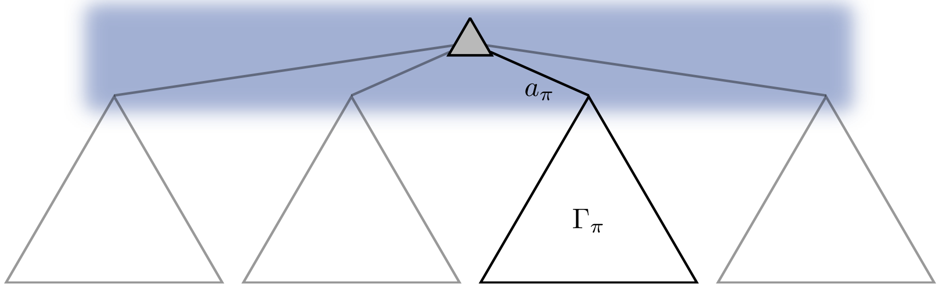
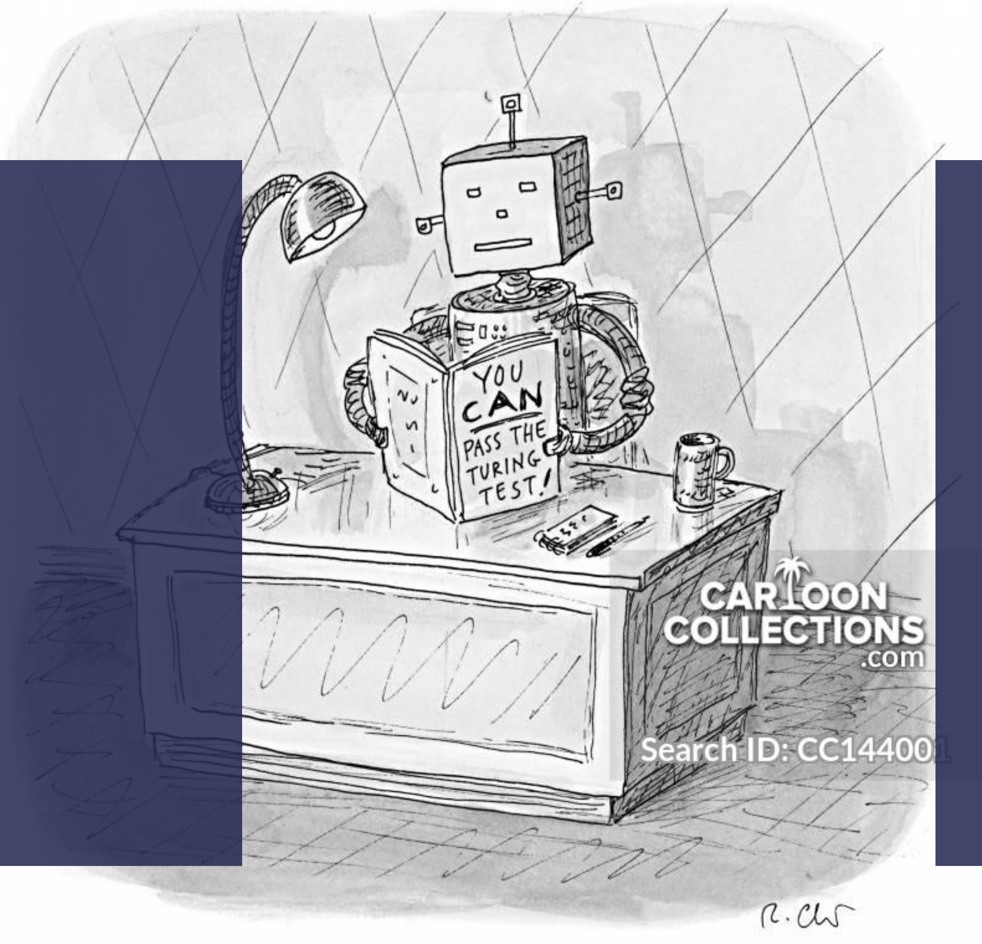


OUR RESEARCH QUESTION

Can we define a better auxiliary game?

We need to keep the game structure

Public Team Conversion procedure



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- ⇒ Team players share a **deterministic strategy of the coordinator** before the game starts
- ⇒ Team players **query** it whenever needed, using the **current state of the game as input**
- ⇒ In order to be shared, the coordinator requires as input **the same information from both players**. Therefore, the **public information state** among the team players is used.

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$\approx$ Localized plan correlation **$\neq$ Ex ante whole plan coordination**

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3. Public Team Conv

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$$Player1\colon\ Raise, Player2\colon\ Call$$

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The resulting game is a **2pOs** game (coordinator vs adversary) that shares the **same public information structure as the original game**

Our main result is **Theorem 2**:

- **The converted game is strategically equivalent to the original one**, since there exists a mapping of equivalent strategies between the original and the converted game
- **A Nash Equilibrium in the converted game is a TMECor in the original game**

Its proof is particularly complex, and is out of the scope of this presentation.

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Additional results regarding the **NP-hardness of the problem vs the potential exponential increase in size** of the converted game due to the **combinatorial structure of prescriptions**:

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Additional results regarding the **NP-hardness of the problem vs the potential exponential increase in size** of the converted game due to the **combinatorial structure of prescriptions**:

- (Farina et al., 2021) proved that finding a TMECor in a **game without private observations** can be done in **polynomial time**. We verified that our conversion procedure does not increase the size of the game in those cases.

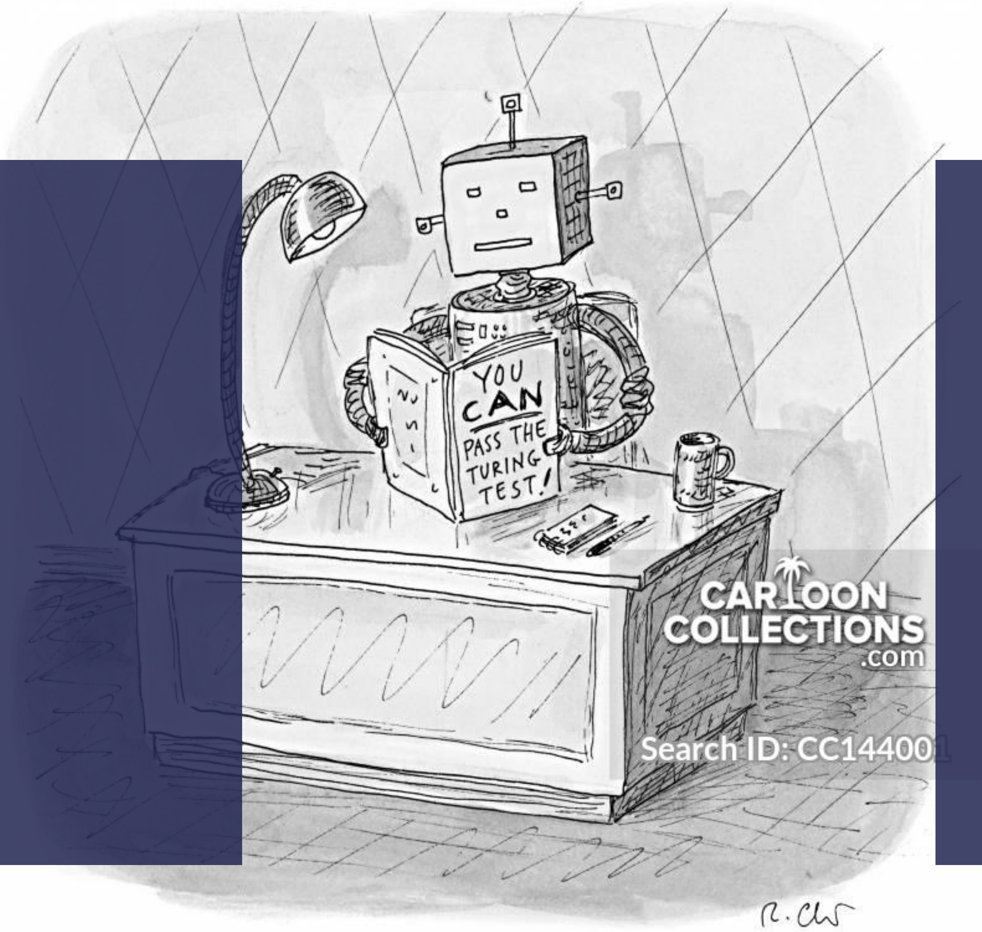
OUR CONTRIBUTIONS

Additional results regarding the **NP-hardness of the problem vs the potential exponential increase in size** of the converted game due to the **combinatorial structure of prescriptions**:

- (Farina et al., 2021) proved that finding a TMECor in a **game without private observations** can be done in **polynomial time**. We verified that our conversion procedure does not increase the size of the game in those cases.

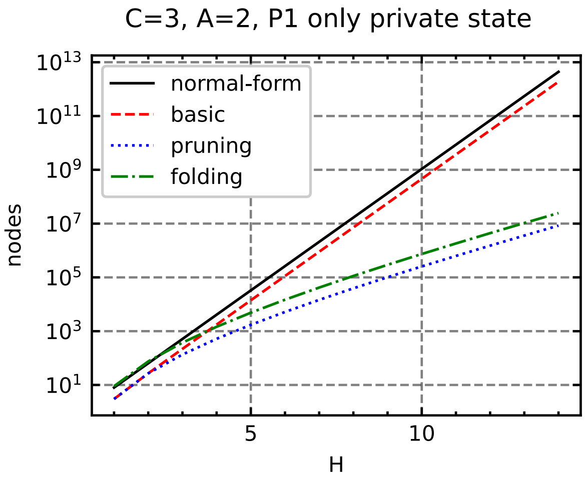
- We developed **three pruning techniques** to mitigate the exponential increase in size of the converted game.
  - **Belief-based pruning**
  - **Folding representation**
  - **Safe Imperfect recall**

Experimental Results

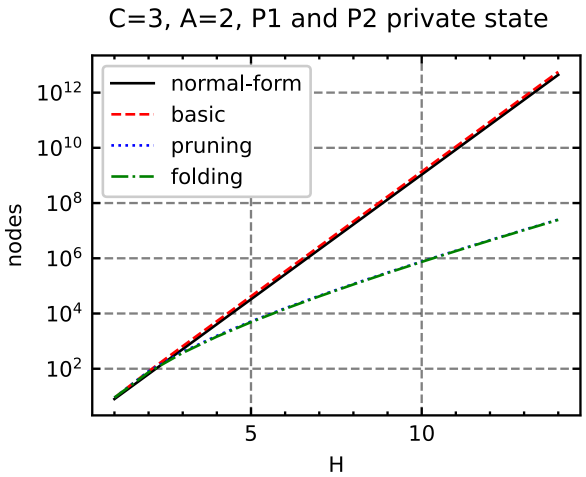


TEST 1 – Impact of Pruning Techniques

We evaluated the impact of the developed pruning techniques on a parametric game with C private states, A actions per node and a variable number of H levels of action



Case when C=3 and A=2, only a single player with private information



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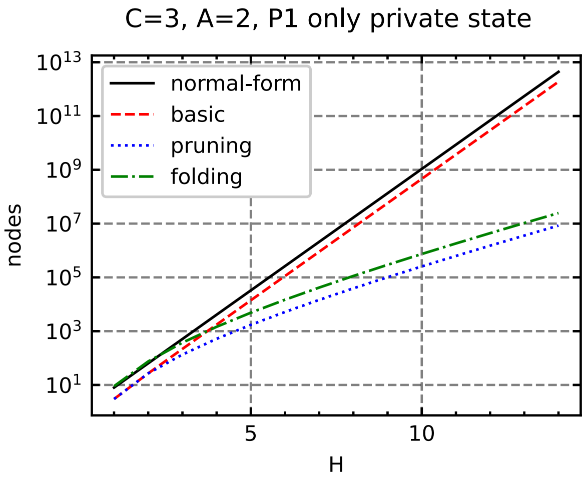

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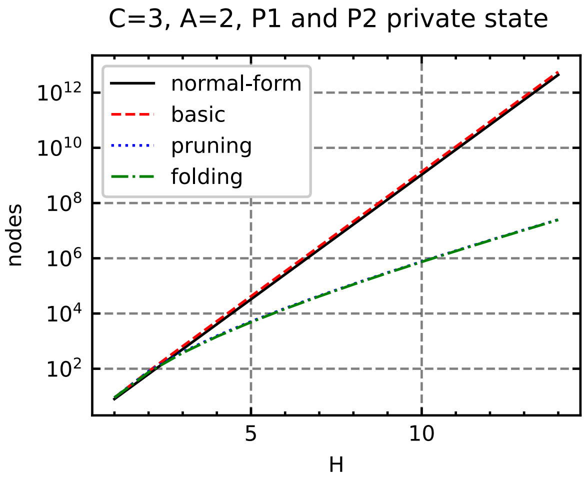

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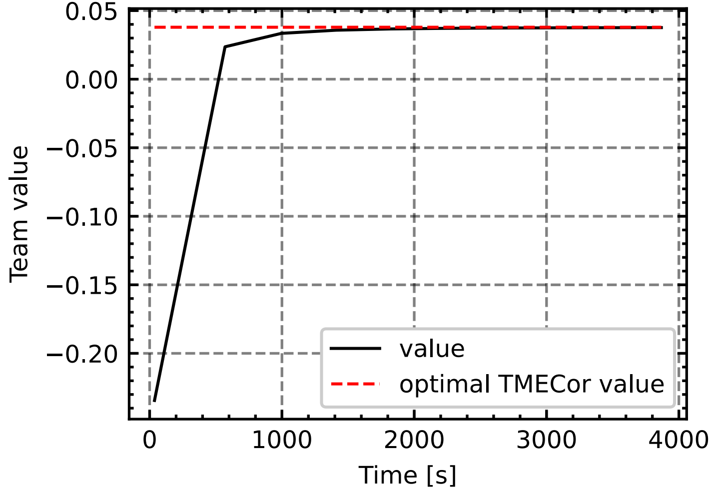
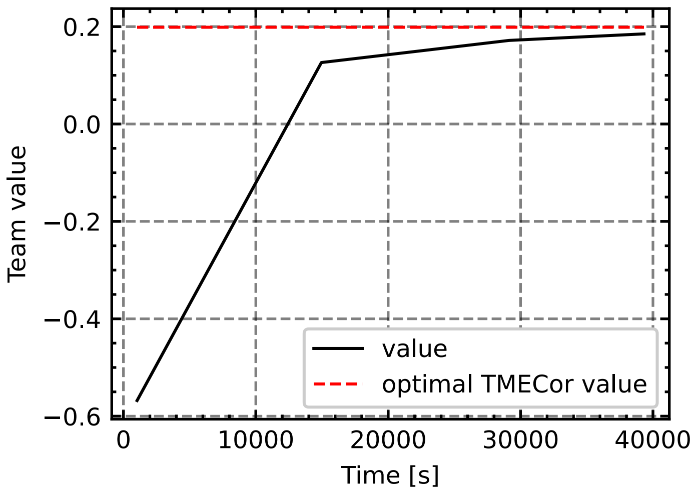
CONCLUSION: we reduce the size of the resulting game up to a square root factor

TEST 2 – Application to Poker instances

We applied standard solving algorithms to the folding representation of Kuhn and Leduc instances



Kuhn Poker, 3 ranks and adversary playing first



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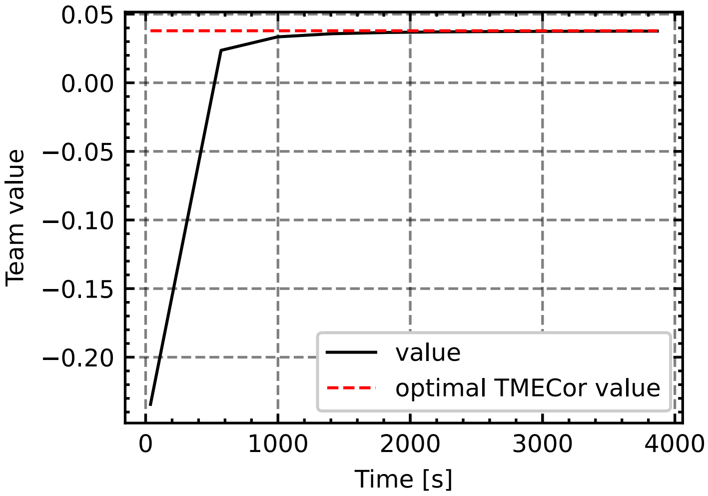

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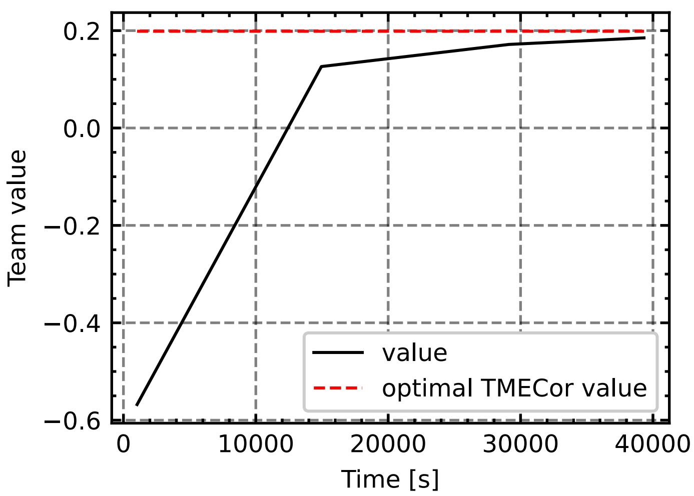
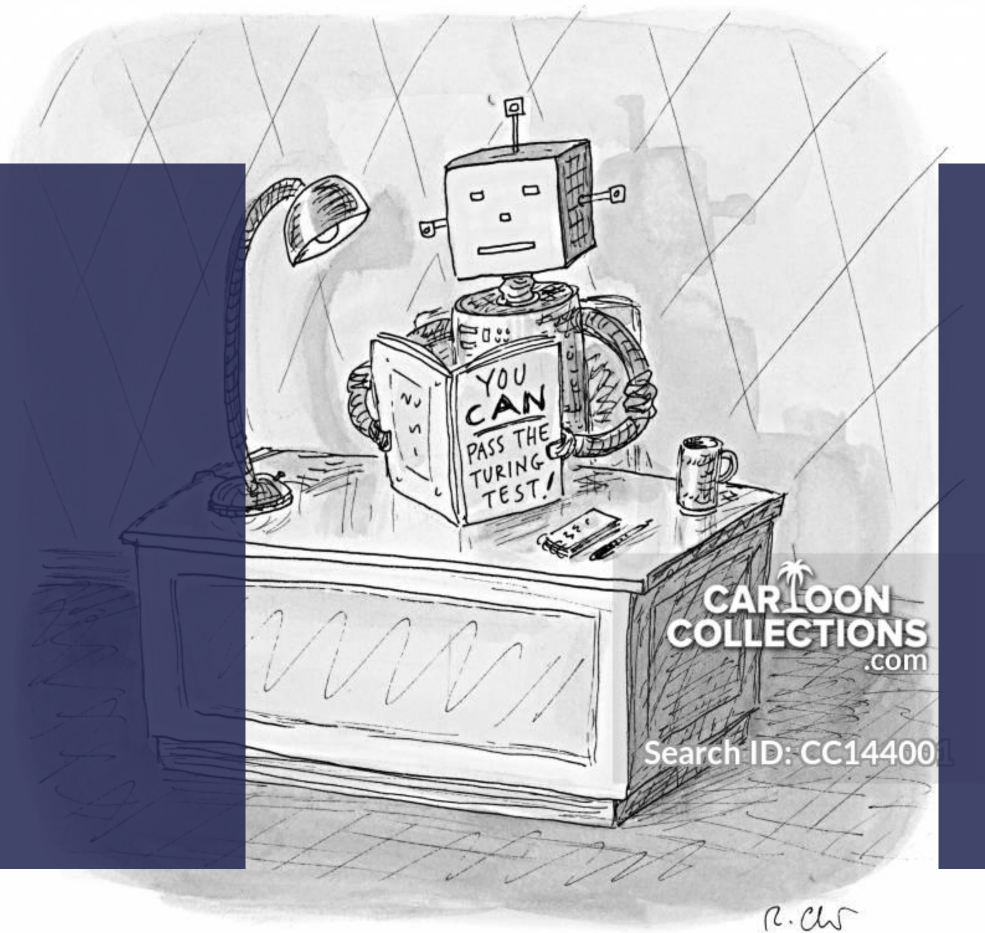


Leduc Poker, 3 ranks, 1 raise, and adversary playing first

CONCLUSION: convergence in value to a TMECor is achieved, coherently with our theoretical result

Implications and Future Works



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The $\textsc{PublicTeamConversion}$ procedure is **recursive**; this allows to avoid the complete construction of a new game, and enables the online generation of trajectories in the converted game

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Since our converted game is a 2p0s game presenting a sequential structure, the framework fo Abstraction and Continual resolving can be applied to potentially allow the development of techniques for larger instances of the game

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Extension to N vs N games

From a theoretical point of view, the idea of a shared coordinator sending prescription to team members can be extended also in the case in which two teams of agents are interacting one against the other

FUTURE WORKS

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Better algorithms to cope with the asymmetry of coordinator vs adversary

The converted game is highly asymmetrical in size, with most of the tree occupied by actions of the coordinator. Our idea is that of tweaking solving algorithms to take advantege of this situation.

Thank you for your attention!
Any Questions?

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