Research Project Proposal: Learning correlated equilibria in constrained normal form games

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1. Introduction to the problem

1.1. Description of the areas and research topics in which the problem is positioned

The research topic is positioned in the three following main areas.

- Game Theory: it is a branch of applied Mathematics mainly used to model the strategic interaction between different players in a context with predefined rules and outcomes.
- Machine Learning: in recent years, learning algorithms have been widely used in the context of Game Theory giving the tools and the possibility to approach new and harder problems.
- the third one is Theoretical Computer Science: in this context, it is of great importance to know the complexity and the difficulty of the problems we are dealing with.

From the union of these three areas Algorithmic Game Theory was born.

1.2. Motivations to support the importance of the research topic

Algorithmic Game Theory is a new and challenging research topic. It involves Mathematics and Computer Science and it requires their knowledge to apply mathematical formulations and algorithms. However, it is not only fun and challenging, it is widely used in a huge number of contexts: economic models, military strategy, networking and many others. Considering also the novelty of the field, it contains a lot of unexplored paths to be followed.

1.3. Description of the problem

The main problem we are going to solve is related to constrained games in normal form. In the next sections further details will be provided. The problem has been already partially analyzed and explored from the point of view of the existence of an equilibrium, but the main contribution we hope to give is from the computational point to view, in finding an efficient learning algorithm in that context.

1.4. Motivations to support the importance of the problem

The presence of some constraints in a strategical situation in which Game Theory is applied is frequent. Some examples follow.

- Budget games: the players need to learn an optimal strategy but, during the learning phase, he cannot spend more than a fixed budget for the exploitation.
- Routing problems: the players need to find the optimal path to the goal under some constraints. These problems can be applied in networking or in route suggestions (i.e. Google Maps).
- Security problems: the objective is to deploy limited security resources to maximize their effectiveness.

2. Main related works

Argyrios Deligkas, John Fearnley & Rahul Savani analyze, in [2], the difficulty of finding a Nash equilibrium and try to find a way to compute an approximate one. In our work, the intention is not to focus on Nash Equilibria: it is known that finding a costrained Nash equilibrium is a PPAD problem and to find an optimal one is NP-Hard. The main focus is to analyze some alternative notion of equilibrium, such as Correlated Equilibrium, or Φ equilibrium. In this context A Greenwald, A Jafari, C Marks, in [3], analyze the notions of Internal External and Φ Regret through Φ equilibria. Finally, a similar objective has been chased by M. Bernasconi, F. Cacciamani, M. Castiglioni, A. Marchesi, N. Gatti and F. Trovò, in [1]. However, the starting point is different: the game is formalized in a Tree-Form, not in a normal form, and this context leads to a much more challenging scenario.

3. Research plan

3.1. Describe the goal of the research

The main goal of my research is to study constrained normal-form games and to find an efficient learning algorithm that leads to an alternative concept of equilibrium with respect to Nash Equilibrium. The general context is a normal-form game with n agents. Each agent has a certain number of actions and, at each round, plays a probability distribution on these actions that we denote as his strategy s_i. At each turn, depending on the actions played by each agent, a utility u and a cost c are assigned to each agent. It is important to underline that the utility of an agent depends on the combination of all the actions played in a round; instead, the cost of an agent depends only on his own strategy. The goal of each player is to find the optimal strategy in order to maximize some concept of utility that, depending on the scenario, can be related to the single agent if the agents play one against the other, or to all the agents if they cooperate to reach a common goal. At the same time, the players have to respect some constraints, one for each of them. The constraints depend on the costs c.

3.2. Describe the nature of the research: theory, application, implementation, experimental, hybrid

The nature of the research is mainly theoretical. The overall objective is to develop algorithms that are efficient in the context of constrained normal form games. However, their applicability can include a lot of practical fields. Then, the obtained implementations will also be, if possible, tested on existing benchmark problems.

3.3. Describe the tasks in which the research is decomposed, remarking the output of each task

The first task is to analyze the context in which we are working more deeply, to characterize it thoroughly and to have a better understanding in order to be able to perform the following steps in the most aware way possible. Then, the next step will concern the development of the first learning algorithm in the context described above. The result of this phase will be the developed algorithm that will be tested through some simulators or through some existing benchmarks. After this step, the work will continue trying to remove some constraints and to make the problem more challenging. For instance, one possibility is to increase the number of constraints for player or to generalize to the case in which the costs depend on all the actions played in a certain round. New algorithms will be developed or the previous one will be adapted and then tested again. After a final solution has been developed, the whole process will be documented and the validity of the obtained results will be checked.

3.4. Provide a simple GANTT diagram of the task

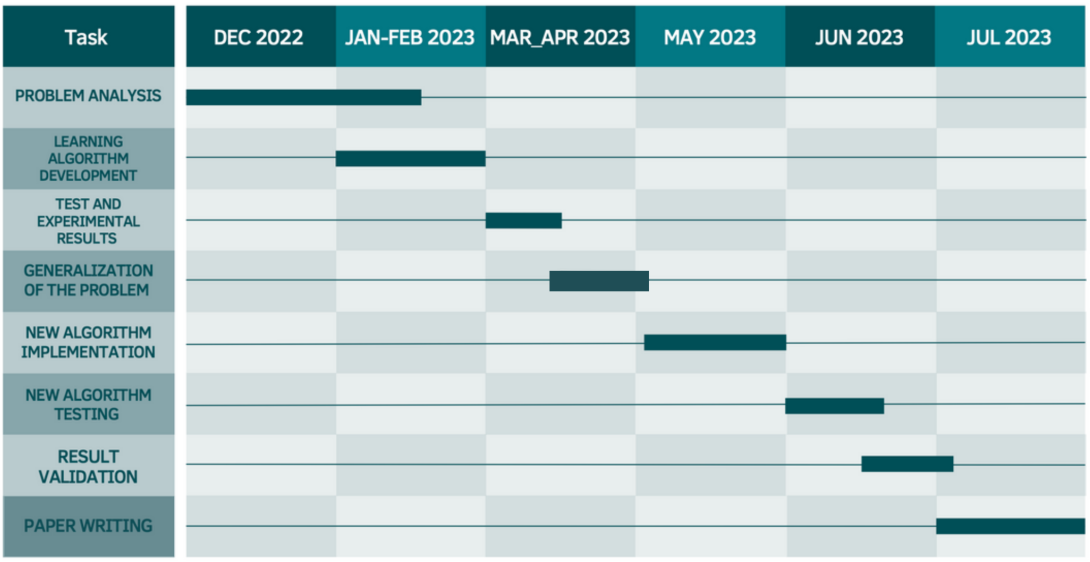

Figure 1: GANTT diagram of the work distribution

3.5. Describe the metrics to use to evaluate the outputs of the research

The metrics used to evaluate the results are mainly formal proofs or experimental results on some existing benchmarks. In some cases there is also the possibility to evaluate the algorithms through some solvers.

References

[1] Bernasconi, M., Cacciamani, F., Castiglioni, M., Marchesi, A., Gatti, N., and Trovò, F. Safe learning in tree-form sequential decision making: Handling hard and soft constraints. In *International Conference on Machine Learning* (2022), PMLR, pp. 1854–1873.

[2] Deligkas, A., Fearnley, J., and Savani, R. Computing constrained approximate equilibria in polymatrix games. In *International Symposium on Algorithmic Game Theory* (2017), Springer, pp. 93–105.

[3] Greenwald, A., Jafari, A., and Marks, C. No-ϕ-regret: A connection between computational learning theory and game theory.