State of the Art: Learning correlated equilibria in constrained normal form games

Alessandro Verosimile, alessandro.verosimile@mail.polimi.it

1. Introduction to the research topic

1.1. Description of the areas in which the research topic is positioned

The research topic is positioned in the three following main areas.

- Game Theory: it is a branch of applied Mathematics mainly used to model the strategic interaction between different players in a context with predefined rules and outcomes.
- Machine Learning: in recent years, learning algorithms have been widely used in the context of Game Theory giving the tools and the possibility to approach new and harder problems.
- the third one is Theoretical Computer Science: in this context, it is of great importance to know the complexity and the difficulty of the problems we are dealing with.

From the union of these three areas Algorithmic Game Theory was born. Among these areas, the main focus of the author will regard the second one and, more precisely, the developing of efficient algorithms capable to learn optimal strategy in specific contexts.

1.2. List of the most prestigious journals and conferences related to the research topic (and the criteria according which such venues are prestigious)

As explained before, the proposed research topic is involved in more than one theoretical area. Then, the number of journal and conferences that can be related to it is really large. The subsequently mentioned ones have been chosen considering two main factors: the relevance in the context of the proposed topic and the quality of the venue, in terms of the Hirsch Index [1]. For what concerns the Journals, the most relevant ones are the following: AIJ, Artificial Intelligence Journal; JMLR, Journal of Machine Learning Research; JAIR, Journal of Artifical Intelligence Research. For what concerns the Conferences, the most relevant ones are the following: Neurips, Neural Information Processing Systems; ICML, International Conference of Machine Learning; AAAI, Association for the Advancement of Artificial Intelligence; IJCAI, International Joint Conference on Artificial Intelligence; AIStats, Artificial Intelligence and Statistics Conference.

1.3. Preliminaries

1.3.1 Mathematical/modeling/algorithmic tools necessary for the understanding of the research topic

In the context of Game Theory we are going to introduce some of the concepts that will be widely used in the research path.

[1]see https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2397014/ for information about the Hirsc Index

Normal Form Games The most basic and useful concept is the representation of a game. The one that will be used to study the research topic is the Normal Form representation. It is based on a tuple (N, A, u), where: N, is the set of players, indexed by i; A is the Cartesian product $A_1 \times ... \times A_n$, where A_i is the finite set of actions of player i. Each vector $a = (a_1, ..., a_n) \in A$ is an action profile; u is the vector $(u_1, ..., u_n)$ where $u_i : A \mapsto \mathbb{R}$ is a real-valued utility function for player i. [6] In the context of the constrained normal form games, the core of the research project, is useful to introduce also the concept of cost. In our initial setting, the costs for player i depend only on the actions of the player i. Then, we can add to the tuple of the game the element: $c = (c_1, ..., c_n)$ where $c_i : a_i \mapsto \mathbb{R}$ is a real-valued cost function for player i.

Strategies A strategy for player i is a probability distribution over the set of m available actions: $s_i = (s_1, ..., s_m)$. A strategy is pure if player i plays an action a with probability 1 and all the others with probability 0, and so it is deterministic. Otherwise, the strategy is said mixed.

Equilibria The strongest concept of equilibrium in Game Theory is Nash-Equilibrium. A strategy profile $S = (s_1, ..., s_n)$ is a Nash equilibrium if, for every agent i, s_i is a best response to s_{-i} [6]. Intuitively, none of the agents can improve his expected payoff deviating on another strategy. Unfortunately, this concept of equilibrium is too strong and it is shown that to find it is a PPAD problem, and to find the optimal one is an NP-Hard problem. Then, some weaker concepts of equilibrium can be used in our context. The most known alternative concepts of Equilibrium are Correlated Equilibrium and Coarse Correlated Equilibrium. They imply the use of a joint mixed strategy to converge to a solution, as there is a coordinator that suggests the action to play to each agent. Some references to Correlated equilibrium and Coarse Correlated equilibrium can be found at [6] and [3], respectively.

1.3.2 Implementation and technological tools used in the field

The purpose of the research topic is to find, in the above described context of constrained normal form games, a Correlated Equilibrium or a Coarse Correlated Equilibrium. Then, the implementation will concern the development of an algorithm which will reach such an objective. The main work will need only the capability of understanding the problem settings and developing new ideas for the implementation; their robustness will be proved through formal mathematical proofs. Some tools can be used during the implementation to test the efficiency of the developed algorithms, (i.e. solvers such as Ampl or Gurobi). In the end some tests on existing benchmarks will be performed. In order to do this Python will be used.

1.4. Description of the research topic

As briefly mentioned before, the main goal of the research is to find an equilibrium in the context of constrained normal form games. In order to better understand the focus of the research, we can visualize the strategy s of a player as a polytope in a m-dimensional space, where m is the number of actions of the player (i.e. in a two actions game it is a triangle, in a three actions game it is a tetrahedron, and so on). The idea of introducing constraints in this setting is equivalent to adding one or more hyper-planes which cut the hyper-space of the strategies. The consequence of this is that some mixed and pure strategies of the hyper-space become unfeasible. This represents, in the most understandable way, the context for what concerns a single agent. Then, the objective is to find a learning algorithm which finds an equilibrium in a game with n agents, each with the above described setting.

1.5. Motivations to support the importance of the research topic

The presence of some constraints in a strategical situation in which Game Theory is applied is frequent. Some examples can be budget games, routing problems or security problems. For a deeper description of these, refer to the Research Project Proposal.

2. Main related works

2.1. Classification of the main related works

2.1.1 Provide and describe some dimensions to classify the main related works

The main dimensions to classify the main related works are:

- the representation form of the game;
- the kind of equilibrium that is reached (i.e. Nash, Correlated, Coarse Correlated or others);
- the number of players (two or n)
- the presence of cost constraints in the game setting.

2.1.2 Provide the classifications of the main related works, remarking which problems are open and which, instead, are fully assessed

The main related works are the following:

- [5] is related to Fisher models in the economic field. They approach an ϵ-approximated equilibrium; the number of players is not fixed; the cost constraints are present; the game is neither represented in normal form nor in extensive form, but it is directly approached as an LP.
- [2] is related to Polymatrix constrained games. They approach an approximated Nash equilibrium; the number of players is not fixed; the cost constraints are present; the game is represented in Polymatrix form.
- [1] is related to learning an optimal strategy in tree-form sequential decision making. The objective is to find an optimal strategy with sub-linear regret; the cost constraints are present; the game is represented in tree-form.
- [4] analyzes the concept of Φ equilibrium and Φ regret, finding the relations with internal and external regret. They approach a Φ equilibrium; the constraints are not present. The game is in normal form.

Equilibrium	Normal Form	Constrained Game
[5] ϵ-approximated	No	Yes
[1] optimal safe strategy	No	Yes
[4] Φ equilibria	Yes	No
[2] Nash-approximated	No	Yes

Table 1: Main related works

2.2. Brief description of the main related works

In the development of my work, the main inspiration comes from two opposite sides: the introduction of constraints that need to be respected by the players, which mainly derives from three works. The first [5] is mainly related to the development of algorithms for computing Fisher's market clearing prices. In their work, they find the first strongly polynomial time algorithm for computing an equilibrium for the linear utilities case of Fisher's market model. This work represent a first important inspiration for their attempt to consider constraints in a game setting. However, it presents important differences: firstly the specific domain to which it is related, secondly the representation of the game which is directly formalized as a linear programming. The work [1], instead, is related to constrained tree-form sequential games. In their setting, an agent sequentially interacts with a stochastic environment defined by means of a tree structure. The agent repeatedly faces the environment over time, and, after each round, it perceives a utility and a cost, which are both stochastic. The goal of the agent is to learn an optimal strategy while keeping costs below a given safety threshold at the same time. Finally, the work [2] studies constrained approximate Nash equilibria in polymatrix games. They show that it is NP-hard to decide if a polymatrix game has a constrained approximate equilibrium for 9 natural constraints and any non-trivial ϵ. All these works are related to the formalization of the game that includes constraints that influence the strategies of the players. For what concerns the formalization of the learning phase, the main inspiration has been the work [4]. In this paper they explore a fundamental connection between computational learning theory and game theory through a property called no-Φ-regret; they deeply analyze the existence of no-Φ-regret algorithms and the possibility of reaching a class of equilibria called Φ-equilibria. The concept of stochastic modification ϕ, the introduced notion of Φ-regret as a generalization of internal and external regret proved to be really useful in the context of regret-minimization and influence the direction of the work. However, they introduce all these notions in an unconstrained setting and they need to be adapted in the actual work setting.

2.3. Discussion about main open issues

In normal form unconstrained games, almost all the possible aspects have been analyzed and widely studied. In the constrained ones, there are more open paths to think about. In the economic fields, through Fisher models, constrained situations have been analyzed, but never in normal form games; also in the study of Bernasconi et al. [1] they have been studied, but in a tree sequential form. In normal form the concept of existence of an approximate equilibrium has been analyzed, but, as far as I know, the learning point of view in this context is an open problem. For what concerns the type of equilibrium we intend to approach, the main focus will not be on Nash Equilibrium: it has been shown that find a Nash equilibrium in a constrained game is PPAD-complete, to find the optimal one is NP-hard. Then, to study the problem in order to approach other concepts of equilibrium such as Correlated or Coarse Correlated equilibrium will surely be more interesting and stimulating.

References

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